Real Wages and the Business Cycle: Accounting for Worker and Firm Heterogeneity*

Anabela Carneiro

Universidade do Porto and CEF.UP

Paulo Guimarães

University of South Carolina, CEF.UP and IZA Bonn

Pedro Portugal†

Banco de Portugal and Universidade NOVA de Lisboa

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Abstract

Using a longitudinal matched employer-employee data set for Portugal over the 1986-2005 period, this study analyzes the heterogeneity in wages responses to aggregate labor market conditions for newly hired workers and existing workers. Accounting for both worker and firm heterogeneity, the data supports the hypothesis that entry wages are much more procyclical than current wages. A one-point increase in the unemployment rate decreases wages of newly hired male workers by around 2.8% and by just 1.4% for workers in continuing jobs. Since we estimate the fixed effects, we were able to show that unobserved heterogeneity plays a

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†Address: Avenida Almirante Reis, 71, 1150-165 Lisboa, Portugal; tel.: 351 21 3128410; e-mail: pportugal@bportugal.pt.

non-trivial role in the cyclicality of wages. In particular, worker fixed effects of new hires and separating workers behave countercyclically, whereas firm fixed effects exhibit a procyclical pattern. Finally, the results reveal, for all workers, a wage-productivity elasticity of 1.2, slightly above the one-for-one response predicted by the Mortensen-Pissarides model.

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1 Introduction

The cyclical behavior of real wages has been the subject of a large number of studies since the debate of Keynes (1939), Dunlop (1938), and Tarshis (1939). Earlier studies based on aggregate data showed some ambiguous results. In this case, the best conclusion is that the choice of the time period analysis, price deflator, and cyclical indicator, as well as the choice between wage rates and average earnings (including overtime or not), may substantially affect the estimates of real wage cyclicality [Abraham and Haltiwanger (1995)]. One reason why these studies have reached no definitive conclusions resides in the fact that they have been performed at the aggregate level. In particular, they have ignored the changes in the composition of the workforce over the cycle. The presence of compositional effects has attracted much attention in the last years and recent micro-data studies based on panel data for the U.S. showed that composition bias plays an important role on real wage behavior along the business cycle [see, for example, Mitchell *et al.* (1985), Bils (1985), Keane *et al.* (1988) and Solon *et al.* (1994)]. In fact, cyclical changes in the composition of the work force may induce a countercyclical bias in the aggregate real wage. Aggregate measures of real wages tend to give more weight to low-skill workers during expansions than during recessions. The argument is that if less-skilled workers are more vulnerable to layoff, they will account for a smaller share of employment in recessions than in expansions. An additional general problem of aggregation is that it assumes that the relationship between real wages and the business cycle is the same for all individuals or groups of individuals. If wrong, the estimates of real wage cyclicality include a specification bias.

Over the last two decades, a number of studies based on micro-panel data for the U.S. (and recently for Britain) point quite decisively toward a procyclical behavior of real wages.[1] Panel microdata also show that real wage changes of job movers are much more procyclical than real wage changes of job stayers [see Solon *et al.* (1994), Shin (1994) and Devereux (2001) for the U.S. and Devereux and Hart (2006) and Hart (2006) for Britain].

Several theoretical explanations have been advanced in order to explain why job changers have more procyclical wages. The more frequent explanation relies on the existence of interindustry wage differentials. This interpretation was first advanced by Okun (1973), who argued that certain jobs offer rents to workers. If these sectors are also more cyclically sensitive, workers can switch into

[1] For insightful surveys see Brandolini (1995) and Abraham and Haltiwanger (1995).

high-paying jobs during booms because such jobs are less tightly rationed during these times.

Beaudry and DiNardo (1991) advanced a more convincing explanation for the differences in wage cyclicality between job stayers and job changers, even though their explanation abstracts from heterogeneity across jobs. According to their findings, current unemployment rate does not affect wages after controlling for the best labor market conditions since a worker was hired at his/her current job. Indeed, when workers are not mobile between employers, current labor market conditions do not affect current wages. In this case, current wages are negatively correlated with the unemployment rate at the time each worker was hired. However, if workers are very mobile, wages are correlated with the best labor market conditions observed since the worker was hired.

Barlevy (2001) offered a new explanation for the existence of more procyclical wages of job changers: compensating differentials. In order to show that compensating differentials instead of interindustry wage differentials generate a more procyclical behavior of wages of changers, Barlevy developed a model that relates unemployment insurance and wage cyclicality. His empirical finding of a negative relationship between wage cyclicality among job changers and the level of unemployment insurance benefits, supports the view that job changers' wages are more procyclical because in booms they obtain jobs that pay a compensating differential for the risk of layoff. In this case, workers who change jobs during booms may not realize true gains from the higher wages they receive, since these gains are typically offset during recessions.

Recent microeconometric evidence on wage cyclicality also gave a new insight to the discussion about business cycle fluctuations of unemployment and vacancies and wage stickiness. Indeed, some authors argue that the Mortensen-Pissarides [Mortensen and Pissarides (1994) and Pissarides (2000)] search and matching model cannot explain the cyclical volatility of unemployment and vacancies [Hall (2003) and Shimer (2005)]. Furthermore, they also show that if the hypothesis of rigid wages is introduced the model performs much better to match fluctuations in unemployment and vacancies.

In a recent exercise, however, Pissarides (2007) showed that the wage stickiness hypothesis does not seem to match the empirical data. Exploring the idea that in the search and matching model job creation is driven by the difference between the expected productivity and the expected cost of labor in new matches, Pissarides shows that in equilibrium the wages negotiated in new matches are about as cyclical as productivity. This prediction of the model seems to be consistent with the empirical evidence that wages in new matches are much more procyclical than wages in continuing jobs.

Haefke *et al.* (2007) also defend this point of view. Using the Current Population Survey (CPS) they showed that wages of newly hired workers are much more volatile than aggregate wages and respond one-to-one to changes in labor productivity.

In this context, the motivation to empirical research is to have appropriate data that allow to test if wages in new matches are more volatile than those in continuing jobs. As mentioned before, previous empirical studies have been showing that job changers' wages are much more procyclical than job stayers' wages. However, and since these studies do not seem to fully control for compositional effects, it can always be argued that the empirical evidence merely reflects the impact on wages of workers drifting from low wage firms to high wage firms in expansions, and vice-versa for recessions.

This paper adds to the empirical literature on wage cyclicality in several ways. The main contribution is to analyze the impact of the cycle on real wage growth of new hires versus stayers within the same firm. The key question to be answered is: are starting-wages, conditional on the long-term wage policy of the firm, more sensitive to the economic cycle? To the best of our knowledge, this is the first study that explicitly deals with this issue controlling simultaneously for worker and firm unobserved heterogeneity which allows handling of both sources of composition bias in wage cyclicality. In fact, beyond the possibility that average worker quality may change over the cycle, for several reasons job quality may also exhibit a cyclical pattern. Some authors provided evidence that in recessions individuals take lower-paying jobs that dissolve quicker whereas in expansions firms create high-paying jobs that last longer (see, for example, Beaudry and DiNardo (1991) and Bowlus (1995)). Furthermore, the industry composition may also change over the cycle. This study claims the importance of controlling for both worker and firm unobserved heterogeneity when analyzing the cyclical behavior of wages. Worker and firm unobserved heterogeneity, both respond strongly to changes in unemployment rates.

For this purpose a unique and rich matched employer-employee longitudinal data set - *Quadros de Pessoal* - will be used and a new iterative procedure which provides the exact OLS solution to the two-way fixed effects model will be employed.

Two additional contributes of this paper deserve attention. The first is to test if the impact of the unemployment rate on wages really reflects labor-market tightness disentangling between the job finding probability and the job separation probability. Finally, we analyze how the two components of observed wages - bargained wage and the wage cushion - evolve over the cycle.

This study will be organized as follows. Section 2 presents the architecture of the Portuguese wage setting system. In Section 3 the data set and methodology are described. The main results and some robustness checks are discussed in Section 4. Conclusions are outlined in Section 5.

2 The Architecture of the Portuguese Wage Setting System

2.1 Collective Bargaining

The Portuguese Constitution provides the juridical principles of collective bargaining and grants unions the right to negotiate. The effects of the agreements are formally recognized and considered valid sources of labor law.

Concerning the bargaining mechanisms, a distinction should be made between the *conventional* regime and the *mandatory* regime. Conventional bargaining results from direct negotiation between employers' and workers' representatives. A mandatory regime, on the other hand, does not result from direct bargaining between workers and employers, being instead dictated by the Ministry of Labor. The Ministry can extend an existing collective agreement to other workers initially not covered by it or it can create a new one, if it is not viable to extend the application of an existing document. A mandatory regime is applied when workers are not covered by unions, when one of the parties involved refuses to negotiate or bargaining is obstructed in any other way.[2] Therefore, the impact of collective bargaining goes far beyond union membership and the distinction between unionized and non-unionized workers or firms becomes meaningless.

Usually, collective negotiations are conducted at the industry or occupation level. Firm-level negotiation, which for a time was a common practice in large public enterprises, has lost importance. The law does not establish mechanisms of coordination between agreements reached in different negotiations; however preference is given to vertical over horizontal agreements, and the principle of the most favorable condition to the worker generally applies.

Since most collective agreements are industry-wide, covering companies with very different sizes and economic conditions, their contents tend to be general, setting minimum working conditions,

[2] Beyond the existence of compulsive extension mechanisms, voluntary extensions are also possible, when one economic partner (workers' representative or employer) decides to subscribe to an agreement which it had initially not signed.

in particular the base monthly wage for each category of workers, overtime pay and the normal duration of work. Moreover, only a narrow set of topics is updated annually, and therefore the content of collective agreements is often pointed out as being too immobile and containing little innovation.

Whatever the wage floor agreed upon for each category of workers at the collective bargaining table, firms are free to pay higher wages, and they often deviate from that benchmark, adjusting to firm-specific conditions [see Cardoso and Portugal (2005)].

The Portuguese system of industrial relations apparently presents features of a centralized wage bargaining system. Indeed, massive collective agreements, often covering a whole industry, predominate in the economy, while firm-level collective bargaining covers a low proportion (less than 10 percen) of the workforce. Moreover, trade union confederations, employers' federations and the Government meet at the national level each year to set a guideline for wage increases (the so-called "social concertation"). However, this guideline is not mandatory and merely guides the collective bargaining that follows. The Council for Social Concertation, latter replaced by the Social and Economic Council, was created in 1984 as a tripartite forum (government, workers and employers' representatives) with the aim of promoting "social concertation", but its role concerning income and wage policies remains limited.

On the other side, the fragmented nature of the trade union structure, the fragmented employers' associations and the multiplicity of bargaining units provides the system with a certain degree of decentralization. Even though collective bargaining in Portugal takes place at a sectorial level and most workers are covered by the bargaining system due to the existence of mandatory extensions, the coordination between bargaining units is rather limited. In fact, the right to negotiate is given upon every employer or employers' association and to every trade union (regardless of the number of affiliated members they represent), and the parties have the possibility of choosing the level of negotiation - regional, occupational, industrial or national. This leads to the existence of a diffuse and complex system of wage bargaining with negotiation fragmented and agreements multiplied.

2.1.1 Minimum Wages

A mandatory minimum monthly wage was set for the first time in Portugal in 1974, covering workers aged 20 or older and excluding agriculture and domestic servants. Currently, there is a unique legal minimum wage that applies to all workers. Workers formally classified as apprentices receive just

80 percent of the full rate.

The minimum wage is updated annually by the parliament, under government proposal.[3] Decisions on the level of the minimum wage are taken on a discretionary basis, usually taking into account past and predicted inflation and after consulting the social partners.

In 2005, the minimum monthly wage level was 374.7 €, representing 40 percent of the average monthly wage in the private sector. In this same year the proportion of workers that received the minimum legal wage was about 5 percen.[4]

3 Data and Methodology

3.1 Data Description

The data source of this study comes from a unique and rich matched employer-employee data set - *Quadros de Pessoal* (*QP*). *QP* is an annual mandatory employment survey collected by the Portuguese Ministry of Labor and Social Solidarity, that covers virtually all establishments with wage earners.[5] Indeed, each year every establishment with wage earners is legally obliged to fill in a standardized questionnaire. Reported data cover the establishment itself (location, industry and employment), the firm (location, industry, employment, sales, ownership, and legal setting) and each of its workers (gender, age, education, skill, occupation, admission date, earnings, and duration of work). The information on earnings is very complete. It includes the base wage (gross pay for normal hours of work), regular benefits, irregular benefits and overtime pay, as well as the mechanism of wage bargaining. Information on normal and overtime hours of work is also available.

Eighteen spells of *QP*, from 1986 to 2005, were available for this study.[6] From 1986 to 1993 the information was collected in March of each year, and since 1994, in October.

There are three main reasons that make this survey a good source for the study of wage cyclicality. The first is its coverage. By law, the questionnaire is made available to every worker in a public space of the establishment. This requirement facilitates the work of the services of the Ministry of Labor that monitor compliance of firms with the law (e. g., illegal work). Indeed, the administrative nature of the data and its public availability implies a high degree of coverage and reliability.

[3] The only exceptions are 1982, when it was not updated, and 1989, when it was updated twice.
[4] Source: Ministry of Labor and Social Solidarity (GEP) - Earnings Survey.
[5] Public administration and non-market services are excluded.
[6] Worker level files are not available for the years of 1990 and 2001.

Currently, the data set collects data on about 350,000 firms and 3 million employees. Second, this survey is conducted on a yearly basis, and its identifying scheme allows accurate identification of firms and workers making it possible to track them over the years. Each firm entering the database is assigned a unique identifying number and the Ministry implements several checks to ensure that a firm that has already reported to the database is not assigned a different identification number. Using this identifier it is possible to pinpoint all firms that have entered and exited economic activity. The workers' identification number is based on a transformation of his/her social security number. We match the individuals over the years based on their identification number, gender, year and month of birth. Finally, this source enables the matching of firms and its workers, which allows us to classify the situation of the worker on the job (stayer/mover, accession/separation). Moreover, employer-reported wage information is known to be subject to less measurement error than worker-reported data.

Our data set includes the population of full-time wage earners in the private non-farm sector, aged between 20 and 55 years old.[7] We have also excluded those individuals for whom some explanatory variable is not available for a particular year, namely those with no information on wages and hours worked. In order to minimize the effects of outliers in wages, we dropped 1 percent of the observations corresponding to the top and bottom tails of the wage distribution.

In order to control for workers' unobserved heterogeneity using a first-differences approach, most empirical studies on real wage cyclicality tend to restrict the sample to workers employed for two consecutive years. In this study this restriction is avoided in order to be able to include in the analysis those individuals with a weak labor force/employment attachment. Thus, our data set includes the individuals that are present in two consecutive years,[8] but also contains those individuals that are present in the QP registers in year t but are absent in year $t+1$ (hereinafter 'separations').[9] It also includes the newly hired workers, the so-called 'accessions'. A worker is classified in period t as newly hired if his tenure in that year is less or equal to 12 months. In this context, a newly hired worker may refer to an individual that moved between firms or to an individual that comes from non-employment or the public sector.

[7] In agriculture a considerable amount of payments are non-pecuniary. We thought it better to exclude these workers from the analysis. In any case, the number of these workers is almost negligible.

[8] It should be noted that when a worker is present in the QP in more than one firm in a given year, then we retain the record for the firm in which the worker had the highest number of hours worked.

[9] Hence, separations are only identified between 1986 and 2004.

Employees present in two consecutive years may also be classified as 'stayers' or 'movers'. A 'stayer' is identified as a worker that was employed within the same firm for two consecutive years. A 'mover' is defined as a worker that moved to a different firm from period $t-1$ to period t.

The male population includes 14,242,814 year×individuals observations corresponding to around 4 million individuals matched by identifying number and date of birth. According to Table 1, male stayers are numerically the most important group corresponding to 10,882,692 observations. Furthermore, 1,203,119 observations refer to movers, 2,410,850 to new hires and 3,445,218 to separations. The female population contains 9,398,893 year×individuals observations corresponding to around 2.9 million individuals. Female stayers correspond to 7,005,185 observations, movers to 672,821, accessions to 1,606,385 observations and separations to 2,415,541. It should be noted that, in a given year, an individual may be classified simultaneously as an accession and separation or, for example, as a mover and a new hire. Thus, the sum of the observations in each of the four groups does not correspond to the total number of observations.

Hereinafter, we will focus our attention on stayers, accessions and separations.

Table 1: Data Set Composition

	Males	Females
Stayers	10,882,682	7,005,185
Movers	1,203,119	672,821
Accessions	2,410,850	1,606,385
Separations	3,445,218	2,415,541

Tables A.1.1 and A.1.2 in Appendix A describe the data for male and female workers, respectively.

3.2 Empirical Methodology

The empirical model that will be used to test for real wage cyclicality is a level wage equation with controls for worker observed and unobserved heterogeneity, firm unobserved heterogeneity and business cycle conditions. The option to define the wage equation in levels is justified by the need to estimate the model for workers' hires and separations since, by construction, panel data are not available in these two cases. Thus, in order to account for worker/firm unobserved heterogeneity the fixed-effects estimator will be used instead of the standard first-differences estimator. This avoids restricting the sample to solely continuously employed workers. The main problem with this procedure is that workers or firms that appear solely once over the entire period of analysis are excluded.[10]

The static form of the model is:

$$\log w_{ift} = \lambda_i + \gamma_f + \alpha_0 t + \alpha_1 t^2 + \mathbf{x}_{ift}\boldsymbol{\beta} + \xi\, cycle_t + u_{ift}$$

where $\log w_{ift}$ is the natural logarithm of the real wage of individual i, in firm f at time t, $cycle_t$ is a cyclical indicator such as the aggregate unemployment rate,[11] t and t^2 are a time trend and its square and $\mathbf{x}_{ift}$ is a vector of time-varying worker characteristics. λ_i is an unobserved worker fixed effect, γ_f a firm-specific effect and u_{ift} is a zero-mean random term with constant variance. Since we are particularly interested in comparing the behavior of real wages over the cycle between stayers, accessions and separations, the model also includes dummy variables for hirings and separations and an interaction term between those dummies and the cyclical indicator.

The coefficient of interest is ξ. If the cyclical indicator corresponds to the unemployment rate, the parameter ξ measures the percent wage change in response to a one-point increase in the unemployment rate. A negative value of ξ implies that wages rise when unemployment diminishes, so that wages are procyclical. If, on the contrary, ξ is positive, wages are countercyclical. As mentioned before, the job finding and the job separation probabilities will also be used as measures of the business cycle.[12] [13]

4 Empirical Results

4.1 Real Wage Sensitivity to the Unemployment Rate

Table 2 reports the estimates of the coefficient of the unemployment rate with respect to wages (ξ) for male workers. Besides the aggregate unemployment rate, each regression includes age (and

[10] Thus, all singletons firm-worker were excluded from the data set, representing around 21% of the total number of observations.

[11] Since wages are set at least six months to one year in advance, there is a delayed relationship between wages and economic growth. To capture this lagged effect we use the unemployment rate of the previous year.

[12] We thank Olivier Blanchard for this suggestion.

[13] In table A.2 of Appendix A the unemployment rate, the job finding probability and the job separation probability are reported for the 1985-2005 period.

its square) as a proxy for labor market experience, a set of dummies for worker's qualification and education levels and a quadratic time trend. The dependent variable is defined as the natural log of real hourly earnings. Hourly earnings correspond to the ratio of total regular payroll and total number of normal hours. Total regular payroll includes base wages, seniority payments and regular benefits. The wages were deflated using the Consumer Price Index (CPI) and are expressed in 1985 Euros.[14]

The standard OLS estimates exhibit a strong procyclical behavior of real wages for all workers. A 1-percentage point (p. p.) decrease in the national unemployment rate raises hourly earnings of male stayers by 3.17 percen, by 3.59 percent for newly hired workers and by 3.5 percent for recently separated workers.

Controlling for firm unobserved heterogeneity leads to a reduction in the semi-elasticities estimates of wages with respect to the unemployment rate, most notably for separations. The results also reveal that real wages of newly hired workers are more responsive to cycle fluctuations than real wages of separating workers or workers in continuing jobs. A 1-percentage point (p. p.) decrease in the national unemployment rate raises hourly earnings of newly hired workers by 3.53 percent and just by 2.94 percent and 2.72 percent, respectively, for stayers and separating workers.

Accounting solely for worker unobserved heterogeneity yields to a further decrease in the semi-elasticities of wages, deepening the difference across stayers/separations and accessions. The semi-elasticities of wages with respect to the unemployment rate are -1.5 percent, -2.73 percent and -1.45 percent for stayers, accessions and separations, respectively. Comparing with previous empirical studies that use micro panel data, we can say that the estimate for stayers lies slightly below the bound of -1.93 obtained by Devereux and Hart (2006) for the U.K..[15][16]

Finally, the wage level equation was re-estimated including both a worker and a firm fixed effect. Estimation of a model with two high-dimensional fixed effects is a non-trivial problem. In Appendix B we discuss our estimation strategy which provides the exact OLS solution.

The unemployment rate coefficient estimates from the two fixed effects model are presented in

[14] Between 1986-93 the price index refers to March of year $t-1$ to March of year t, whereas from 1994 to 2005 the price index corresponds to October of year $t-1$ to October of year t.

[15] Martins (2007), using the data set from *Quadros de Pessoal* for the 1986-2004 period, obtained an estimate for male workers employed in two consecutive years of -0.62. The difference behind these results may reside on the estimation procedure employed by author, who used a two-stage first differences estimator.

[16] For a recent summary of these results see Pissarides (2007).

the bottom of Table 2. These results corroborate our previous findings. First, once worker and firm heterogeneity are accounted for, no significant differences are found in the estimates of the cyclicality of hourly earnings between stayers and separations. Second, entry wages are much more procyclical than current wages. In particular, we found that a 1-percentage point increase in the unemployment rate decreases hourly earnings by 1.41 percent for male stayers and by 2.77 percent for newly hired workers.

Table 2: Real Wage Sensitivity to the Unemployment Rate - Men

Portugal, 1986-2005 (N=11,204,120)

Dependent variable: log of real hourly earnings

	Stayers	Accessions	Separations
OLS estimator			
Cycle variable: Unemployment Rate	-3.17	-3.59	-3.50
Standard errors	(0.014)	(0.039)	(0.030)
Within estimator, firm fixed effects			
Cycle variable: Unemployment Rate	-2.94	-3.53	-2.72
Standard errors	(0.001)	(0.035)	(0.020)
Within estimator, worker fixed effects			
Cycle variable: Unemployment Rate	-1.50	-2.73	-1.45
Standard errors	(0.001)	(0.021)	(0.016)
OLS solution with worker and firm fixed effects			
Cycle variable: Unemployment Rate	-1.41	-2.77	-1.14
Standard errors	(0.005)	(0.002)	(0.011)

4.2 Accounting for Aggregate Uncertainty

In our estimation, we are using the whole population of paid workers in the private sector in the Portuguese economy. In a cross-sectional sense, there is no sampling error to take into account. Irrespective of this largely philosophical discussion, the OLS standard errors are, of course, outrageously low. This procedure, however, does not properly accommodate the aggregate uncertainty, in a conventional time-series sense. After all, one solely observe 18 different values for the cycle variables. We provide two ways to deal with this problem. The first approach uses robust clustered standard-errors, a common solution used to circumvent the presence of aggregate covariates in microeconometric regression models. The second approach uses the results from an aggregated regression.[17][18] This latter approach is in the same spirit of the two-stage procedure used in this literature where, in the first stage, the regression model includes the year dummies and, in the second stage, the estimated coefficients of the year dummies are regressed on the cycle variable.

Table 3: Real Wage Sensitivity to the Unemployment Rate - Men

Accounting for Aggregate Uncertainty

Portugal, 1986-2005

Dependent variable: log of real hourly earnings

	Stayers	Accessions	Separations
OLS solution with worker and firm fixed effects (N=11,204,120)			
Cycle variable: Unemployment Rate	-1.41*	-2.77*	-1.14*
Cluster-robust standard errors	(0.403)	(0.447)	(0.404)
Simple regression on the aggregated variables (N=18)			
Cycle variable: Unemployment Rate	-1.39*	-2.66*	-1.03***
OLS standard errors	(0.448)	(0.401)	(0.495)

Note: * significant at 1%; ** significant at 5%; *** significant at 10%.

From Table 3 one can extract three important results. First, accounting for aggregate uncertainty inflates severely the standard errors. The standard errors increased by a factor of one hundred. Second, the two practical solutions to correct the standard errors provide practically identical outcomes. And three, the estimates of the semi-elasticities produced by the two procedures are very close.

4.3 The Cyclical Behavior of Worker and Firm Heterogeneity

Our estimation procedure produces estimates of the worker ($\hat{\lambda}_i$) and firm fixed effects ($\hat{\gamma}_f$). Those fixed effects capture observed and unobserved constant heterogeneity. Those effects are, obviously,

[17] We thank Manuel Arellano for shaving suggested us this solution.

[18] The implementation details for both approaches are discussed in Appendix B.

constant. The presence or absence of workers and firms over the sampling period, however, may exhibit a cyclical pattern. In Table 4, we look at the cyclical behavior (as measured by the unemployment rate) of the worker and firm fixed effects.[19]

Worker persistent heterogeneity (say, observed and unobserved skills) is mildly countercyclical, in particular, for separating workers. This evidence seems to suggest that during recessions firms hire and fire skilled workers in larger proportions. Firm heterogeneity (meaning say, higher paying firms), however, is procyclical, most notably for newly hired and separating workers. This seems to be in line with evidence that in booms high-paying firms create/destroy jobs in larger proportions than low-paying firms. Overall, for the unemployment cycle variable, compositional bias plays a significant role.

Table 4: The Cyclical Behavior of Worker and Firm Heterogeneity - Men

Portugal, 1986-2005 (N=11,204,120)

Cycle variable: Unemployment Rate

	Stayers	Accessions	Separations
Dependent variable: $\hat{\lambda}_i$	0.104	0.487**	1.073*
Cluster-robust standard errors	(0.112)	(0.205)	(0.215)
Dependent variable: $\hat{\gamma}_f$	0.156	-1.373*	-1.082*
Cluster-robust standard errors	(0.124)	(0.134)	(0.161)

Note: * significant at 1%; ** significant at 5%; *** significant at 10%.

4.4 Real Wage Sensitivity to Aggregate Labor Productivity

An alternative approach to analyze the cyclical behavior of wages, which is more closely rooted in the Mortensen and Pissarides theoretical framework, is to estimate the elasticity of wages with respect to aggregate labor productivity. In order to analyze the reaction of real wages to labor productivity, we replace the unemployment rate by the measure of aggregate labor productivity

[19] These estimates were obtained running an OLS regression of the fixed effect estimates on the unemployment rate and a quadratic time trend.

in the Portuguese private sector, as a cycle variable. The aggregate labor productivity measure is defined as the Gross Domestic Product (GDP) per capita in the private sector. Aggregate labor productivity was deflated using the GDP deflator.[20] Table 5 shows that wages for newly hired workers, workers in ongoing job relationships, and workers leaving their jobs exhibit an elasticity that is not at odds with the theorectical notion that it should be one, that is, a one-for-one wage response to changes in labor productivity. Interestingly, Blanchard (2007) argues that one of the main reasons for the Portuguese macroeconomic imbalances is rooted in the recent evolution of real wages 20 percent above productivity.

In the case of aggregate labor productivity, compositional bias does not play a significant role, at least when one compares the worker within estimates with the two-way fixed effects results.

[20] Table A.2 of Appendix A shows the evolution of aggregate labor productivity over the 1985-2005 period.

Table 5: Real Wage Sensitivity to Aggregate Labor Productivity - Men

Portugal, 1986-2005

Dependent variable: log of real hourly earnings

	Stayers	Accessions	Separations
OLS estimator			
Cycle variable: Aggregate Labor Productivity	0.707	0.713	0.648
Within estimator, firm fixed effect (N=11,204,120)			
Cycle variable: Aggregate Labor Productivity	0.694	0.672	0.667
Within estimator, worker fixed effect (N=11,204,120)			
Cycle variable: Aggregate Labor Productivity	1.306	1.307	1.230
OLS solution with worker and firm fixed effects (N=11,204,120)			
Cycle variable: Aggregate Labor Productivity	1.217*	1.264*	1.185*
Cluster-robust standard errors	(0.061)	(0.060)	(0.061)
Simple regression on aggregated variables (N=18)			
Cycle variable: Aggregate Labor Productivity	1.212*	1.245*	1.169*
OLS standard errors	(0.069)	(0.050)	(0.051)

Note: * significant at 1%; ** significant at 5%; *** significant at 10%.

4.5 Bargained Wage, Wage Cushion and the Business Cycle

Here we examine to what extent contractual wages, on the one hand, and firm-specific wage arrangements, in the form of the wage cushion, on the other, are sensitive to the business cycle. Cardoso and Portugal (2005) showed that in Portugal the wage cushion works as a mechanism to overcome the constraints imposed by collective bargaining, granting firms certain freedom when setting wages. In this context, it will be interesting to analyze to what extent contractual wages and firm deviations from contractual wages vary over the business cycle.

The contractual wage was computed adopting the procedure suggested by Cardoso and Portugal

(2005). Thus, the BARGW was defined as the mode of the monthly base wage for each worker category within each collective agreement. The wage cushion (WCUSH) was computed as the log difference between the current actual wage and the current contractual wage for that professional category. As exhibited in Table 6.A, the bargained wage is very sensitive to the evolution of the unemployment rate, especially for new hires. The wage cushion, however, exhibits a cyclical behavior solely for new hires (see Table 6.B). It seems that firms in expansions are forced to pay starting wages significantly above those settled in sectoral wage agreements.

Table 6.A: Sensitivity of Bargained Wages to the Unemployment Rate - Men

Portugal, 1986-2005

Dependent variable: log of real bargained wage

	Stayers	Accessions	Separations
OLS solution with worker and firm fixed effects (N=11,204,120)			
Cycle variable: Unemployment Rate	-1.93*	-2.18*	-1.67*
Cluster-robust standard error	(0.323)	(0.366)	(0.328)
Simple regression on aggregated variables (N=18)			
Cycle variable: Unemployment Rate	-1.91*	-2.28*	-1.76*
OLS standard errors	(0.358)	(0.330)	(0.415)

Note: * significant at 1%; ** significant at 5%; *** significant at 10%.

Table 6.B: Sensitivity of the Wage Cushion to the Unemployment Rate - Men

Portugal, 1986-2005

Dependent variable: log of real bargained wage

	Stayers	Accessions	Separations
OLS solution with worker and firm fixed effects (N=11,204,120)			
Cycle variable: Unemployment Rate	0.10	-0.67*	0.07
Cluster-robust standard error	(0.165)	(0.195)	(0.142)
Simple regression on aggregated variables (N=18)			
Cycle variable: Unemployment Rate	0.06	-0.58*	0.12
OLS standard errors	(0.136)	(0.224)	(0.184)

Note: * significant at 1%; ** significant at 5%; *** significant at 10%.

4.6 Robustness Checks

4.6.1 Alternative Wage Measures

In order to check if our results are robust to alternative definitions of wages, the model was re-estimated using alternative wage measures. Three other measures of wages were considered: the monthly base wage, the hourly base wage and the hourly earnings including overtime pay. The hourly base wage is defined as the ratio between the monthly base wage and the total number of normal hours worked in the month. The hourly earnings including overtime pay is defined as the ratio between total regular payroll including overtime pay and the sum of normal and extra hours of work. As mentioned above, the wages were deflated using the CPI.

The two-way fixed effects results are presented in Table 7 for male and female workers. For comparison reasons the unemployment coefficient estimates for hourly earnings are reported in the first row.

The results exhibit, regardless the measure of wages used, a slightly less procyclical behavior of women's real wages compared to men's real wages. This empirical evidence is in accordance with previous findings by Tremblay (1990) and Solon *et al.* (1994).

The inclusion of overtime pay does not change much the estimate of the unemployment rate coefficient for either men and women. This result is not surprising since in the Portuguese labor market overtime hours represent a small percentage of total hours worked.[21]

Comparing the figures obtained for the estimates of the unemployment coefficient using a monthly measure instead of an hourly measure, leads us to conclude that monthly wages are more procyclical than hourly wages, suggesting, as should be expected, that hours worked also exhibit a procyclical pattern.

[21] In the 1986-2005 period, overtime work for firms employing paid labor corresponds, on average, to 0.9% of the total number of hours worked.

Table 7: Real Wage Sensitivity to the Unemployment Rate

Portugal, 1986-2005

Alternative Wage Measures

Cycle variable: Unemployment Rate

	Stayers	Accessions	Separations
Men (N=11,204,120)			
OLS solution with worker and firm fixed effects			
Dependent variable			
Hourly Earnings	-1.41	-2.77	-1.14
Cluster-robust standard errors	(0.403)	(0.447)	(0.404)
Hourly Earnings inc OT	-1.39	-2.66	-1.08
Cluster-robust standard errors	(0.400)	(0.389)	(0.413)
Hourly Base Wage	-1.54	-2.80	-1.30
Cluster-robust standard errors	(0.397)	(0.392)	(0.388)
Monthly Base Wage	-1.80	-2.91	-1.58
Cluster-robust standard errors	(0.311)	(0.330)	(0.280)
Women (N=7,293,755)			
OLS solution with worker and firm fixed effects			
Dependent variable			
Hourly Earnings	-1.11	-2.31	-0.75
Cluster-robust standard errors	(0.360)	(0.342)	(0363)
Hourly Earnings inc OT	-1.10	-2.30	-0.73
Cluster-robust standard errors	(0.364)	(0.345)	(0.368)
Hourly Base Wage	-1.34	-2.57	-1.04
Cluster-robust standard errors	(0.379)	(0.350)	(0.374)
Monthly Base Wage	-1.57	-2.53	-1.22
Cluster-robust standard errors	(0.285)	(0.271)	(0.282)

4.6.2 Disentangling between Job Finding and Job Separation Probabilities

In this Section the impact of the business cycle on real wages is analyzed using a different cyclical indicator. In particular, the job finding and the job separation probabilities in period $t-1$ were included in the wage equation as alternative measures to the unemployment rate.[22] The results are shown in Table 8. The coefficients of the new measures have the expected signs: a negative effect on hourly earnings for the job separation probability and a positive one for the job finding probability. Thus, these figures are consistent with the unemployment rate estimates, though their magnitudes cannot be directly compared.

Real wages react both to a changes in the probability of finding a job and to a changes in the job separation probability. For example, a 1 percentage point (p. p.) increase in the job finding probability corresponds to an increase of 0.7 percent on real wages for newly hired workers, whereas an increase 1 p. p. in the probability of job separation corresponds to a real wage decrease 12 percent for newly hired workers. Despite the dissimilitude of the coefficient estimates, these two variables generate real wage fluctuations of identical amplitude.

Finally, these results indicate a less pronounced difference in the behavior of real hourly earnings over the cycle across stayers and accessions.

[22] These probabilities were calculated according to Franco and Torres (2008). The job separation probability is given by $\frac{u^S_{t+1}}{e_t}$, where u^S_{t+1} is the number of short-term unemployed persons in quarter $t+1$ (unemployed for less than three months) and e_t corresponds to the level of employment in quarter t. The job finding probability is given by $\frac{u_t - u_{t+1} - u^s_{t+1}}{u_t}$, where u_t refers to the stock of unemployed persons in quarter t.

Table 8: Real Wage Sensitivity to Job Separation and Job Finding Probability - Men

Dependent variable: log of real hourly earnings

	Stayers	Accessions	Separations
OLS solution with worker and firm fixed effects (N=11,204,120)			
Cycle variable: Job Separation Probability	-0.091*	-0.117*	-0.126*
Cluster-robust standard errors	(0.025)	(0.028)	(0.022)
Simple regression on aggregated variables (N=18)			
Cycle variable: Job Separation Probability	-0.091*	-0.118*	-0.127*
OLS standard errors	(0.035)	(0.033)	(0.034)
OLS solution with worker and firm fixed effects (N=11,204,120)			
Cycle variable: Job Finding Probability	0.004*	0.007*	0.005*
Cluster-robust standard errors	(0.001)	(0.001)	(0.001)
Simple regression on aggregated variables (N=18)			
Cycle variable: Job Finding Probability	0.004*	0.006*	0.005*
OLS standard errors	(0.001)	(0.001)	(0.001)

Note: * significant at 1%; ** significant at 5%; *** significant at 10%.

5 Conclusion

The aim of this study is to provide further evidence on real wage cyclicality using a rich longitudinal matched employer-employee dataset for the 1986-2005 period, addressing the issue of heterogeneity in wages responses to aggregate labor market conditions, across different groups of workers: stayers, accessions, and separations. For this purpose, we employ information on the registry of all Portuguese wage earners in the private sector comprising around 24 million worker-year observations.

The main contribution of this paper is to analyze the impact of the business cycle on real wages accounting simultaneously for worker and firm permanent unobserved heterogeneity. To achieve this objective, we employ a new iterative procedure which provides the exact OLS solution to the two-way fixed effects model. To the best of our knowledge, previous empirical research on wage cyclicality never consider the role of firm heterogeneity, restricting the attention solely to the compositional bias generated by the presence of worker unobserved heterogeneity.

The empirical evidence gathered in this exercise is sixfold. First, accounting for both worker and firm heterogeneity, there is an indication of a moderate procyclical behavior of real wages for stayers. The semi-elasticities elasticities of wages with respect to the unemployment rate are in the order of -1.1 percent for females to -1.4 percent for males. That is, among workers in continuing jobs, an increase of a percentage point in the unemployment rate leads to a real wage decline between 1.1 and 1.4 percent. Identical estimates were obtained for recently separated workers.

Second, wages of recently hired workers are much more procyclical than the wages of continuing employed workers. A one-point increase in the unemployment rate decreases real wages of newly hired workers by around 2.8 percent for men, and 2.3 percent for women. These results are robust to changes in the definition of wages. They seem to vindicate Pissarides (2007) presumption that one needs a semi-elasticity close to 3 to be able to explain the cyclical volatility of unemployment and vacancies within the Mortensen-Pissarides model.

Third, and more directly related to the volatility puzzle debate, we found that wages for all types of workers exhibit an elasticity that is not at odds with the theoretical notion that it should be one, that is, a one-for-one wage response to changes in labor productivity.

Four, compositional bias plays a very important role. Failure to account for worker unobserved (constant observed) heterogeneity may induce a countercyclical bias in wage cyclicality, whereas failure to control for firm unobserved heterogeneity may lead to a procylical bias, at least for new hires and separations.

Fifth, when we employ alternative measures of the business cycle, that is, when we use the job finding probability and the job separation probability instead of the unemployment rate, we again found support for the cyclicality of real wages. In those specifications, however, the distinction between new hires and stayers is less sharp, most notably, for the measure of job separation probability.

And sixth, because firms often pay wages above the wage floors negotiated between the employer associations and the trade unions, we decompose the observed wage between the bargained wage component – that agreed at the bargaining table – and the wage cushion component – that obtained from the difference between the actual wage and the bargained wage. We found that, for all workers, the bargained wage is very sensitive to the evolution of the unemployment rate, whereas the wage cushion exhibits a cyclical behavior solely for newly hired workers.

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APPENDIX A - Descriptive Statistics

Table A.1.1: Descriptive Statistics (1986-2005)

Men (N=11,204,120)

	MEAN	STDV	MIN	MAX
Variables				
Age (in years)	36.4	9.7	20.0	56.0
Education Level				
Less than Basic School	0.025		0.0	1.0
Basic School	0.398		0.0	1.0
Preparatory	0.208		0.0	1.0
Lower Secondary	0.149		0.0	1.0
Upper Secondary	0.145		0.0	1.0
College	0.054		0.0	1.0
Non-defined	0.020		0.0	1.0
Qualification Level				
Top Executives	0.046		0.0	1.0
Intermediary Executives	0.034		0.0	1.0
Supervisors	0.057		0.0	1.0
Highly Skilled and Skilled Professionals	0.569		0.0	1.0
Semi-skilled and Unskilled Professionals	0.221		0.0	1.0
Apprentices	0.041		0.0	1.0
Non-defined	0.032		0.0	1.0
Monthly Base Wage (in real euros)	536.3	381.5	50.7	4061.0
Hourly Base Wage (in real euros)	3.2	2.38	0.4	23.2
Hourly Earnings (in real euros)	3.8	3.14	0.4	194.0
Hourly Earnings inc OT (in real euros)	3.8	3.16	0.4	194.0

Table A.1.2: Descriptive Statistics (1986-2005)

Women (N=7,293,755)

	MEAN	STDV	MIN	MAX
Variables				
Age (in years)	34.4	9.1	20.0	56.0
Education Level				
Less than Basic School	0.024		0.0	1.0
Basic School	0.332		0.0	1.0
Preparatory	0.205		0.0	1.0
Lower Secondary	0.154		0.0	1.0
Upper Secondary	0.191		0.0	1.0
College	0.074		0.0	1.0
Non-defined	0.020		0.0	1.0
Qualification Level				
Top Executives	0.032		0.0	1.0
Intermediary Executives	0.027		0.0	1.0
Supervisors	0.024		0.0	1.0
Highly Skilled and Skilled Professionals	0.472		0.0	1.0
Semi-skilled and Unskilled Professionals	0.351		0.0	1.0
Apprentices	0.069		0.0	1.0
Non-defined	0.025		0.0	1.0
Monthly Base Wage (in real euros)	456.2	324.4	50.4	4050.0
Hourly Base Wage (in real euros)	2.74	2.11	0.35	23.2
Hourly Earnings (in real euros)	3.11	2.61	0.35	118.6
Hourly Earnings inc OT (in real euros)	3.12	2.62	0.35	118.6

Table A.2: Unemployment Rate, Job Finding Probability, Job Separation Probability and Aggregate Labor Productivity

Portugal, 1985-2005

	Unemployment Rate (%)	Job Finding Probability (%)	Job Separation Probability (%)	Aggregate Labor Productivity
1985	7.2	12.4	1.35	4.44
1986	7.4	17.4	1.41	4.61
1987	6.5	21.1	1.30	4.89
1988	5.7	20.3	1.16	5.02
1989	4.5	20.0	1.14	5.28
1990	5.1	25.7	1.24	5.40
1991	4.7	26.4	1.16	5.48
1992	3.9	21.1	1.26	5.51
1993	5.0	15.0	1.40	5.68
1994	6.0	17.5	1.59	5.78
1995	6.2	15.4	1.30	5.85
1996	6.3	15.1	1.23	5.92
1997	5.9	21.5	1.37	6.07
1998	5.0	26.2	1.26	6.18
1999	4.4	25.1	1.24	6.27
2000	3.9	24.4	1.11	6.32
2001	4.0	25.7	1.35	6.29
2002	5.0	19.5	1.71	6.35
2003	6.3	21.6	1.67	6.40
2004	6.8	15.8	1.56	6.54
2005	7.6	16.7	1.57	6.60

Source: Bank of Portugal and Franco and Torres (2008).

APPENDIX B

Estimation of the worker and firm fixed effects model In this appendix we detail our estimation technique for the model that includes both firm and worker fixed effects. Because of the high dimensionality of the problem adding actual dummy variables to account for any of the fixed effects is not a feasible approach. However, as it turns out, application of a partitioned algorithm greatly simplifies the estimation problem at hand and leads to the full least squares solution. The idea consists of estimating the model by cycling between estimation on subsets of the parameters [for a discussion of partitioned algorithms see, for example, Smyth (1996)]. To see how this applies to our problem consider the simplest case of the typical least squares dummy variable model

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{D}\boldsymbol{\alpha} + \boldsymbol{\varepsilon} \ , \tag{A.1}$$

where $\mathbf{X}$ is an $M \times k$ matrix of regressors, $\mathbf{D}$ is an $M \times n$ matrix containing n dummy variables that account for group membership and $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$ are the unknown parameters. If the least squares solution for the $\boldsymbol{\alpha}$ were known then we could obtain the least squares estimates of $\boldsymbol{\beta}$ by regressing $\mathbf{Y}$ on $\mathbf{X}$ and an additional single variable containing the estimates of the αs. On the other hand, if the least squares solution for $\boldsymbol{\beta}$ were known we could easily obtain least squares estimates for the $\boldsymbol{\alpha}$. The normal equations show that these are simply the group means of the elements of the vector $\mathbf{u} = \mathbf{Y} - \mathbf{X}\widehat{\boldsymbol{\beta}}$. Hence, an estimation procedure for $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$ could be implemented as follows:

1) Obtain initial values for $\boldsymbol{\beta}$ by regressing $\mathbf{Y}$ on $\mathbf{X}$;

2) Compute $\mathbf{u}$ using the latest estimates of $\boldsymbol{\beta}$;

3) Calculate estimates of $\boldsymbol{\alpha}$ as the group means of the elements of $\mathbf{u}$;

4) Estimate $\boldsymbol{\beta}$ by regressing $\mathbf{Y}$ on $\mathbf{X}$ and an additional variable containing the last estimates of the $\boldsymbol{\alpha}$;

5) Return to step 2 until convergence.

Notice that computationally all that is required is the estimation of several least squares regressions with $k+1$ explanatory variables and group means of elements of $\mathbf{u}$. As is well known a better approach to estimate $\boldsymbol{\beta}$ consists on running a regression in terms of deviations from group means (the within-groups estimator). Consider now the case when a second fixed effect is added to (A.1),

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{D}\boldsymbol{\alpha} + \mathbf{G}\boldsymbol{\gamma} + \boldsymbol{\varepsilon} \ , \tag{A.2}$$

where $\mathbf{G}$ is a matrix of dimension $M \times p$ containing p columns indicating membership to the second group and $\boldsymbol{\gamma}$ is a vector of parameters. Now, the within-groups estimator is no longer a viable alternative. However, the partition algorithm detailed above can be easily modified to accommodate this more complex case. In this case one iterates between estimation of $\boldsymbol{\beta}$, $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$. To estimate $\boldsymbol{\beta}$ in each iteration we regress $\mathbf{Y}$ on $\mathbf{X}$ and two additional variables, containing the last available estimates of $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$. In each step we obtain estimates for $\boldsymbol{\alpha}$ by computing the group means of $\mathbf{u}_1 = \mathbf{Y} - \mathbf{X}\widehat{\boldsymbol{\beta}} - \widehat{\boldsymbol{\gamma}}$ and the estimates for $\boldsymbol{\gamma}$ are computed in a similar way. Thus, the full least squares equation may be estimated by iteratively running a linear regression with only $k+2$ regressors and computing group means. A well known disadvantage of partitioned algorithms is their slow rate of convergence. To accelerate convergence of the model with two fixed effects we adopt two strategies. First, we eliminate the need to estimate the second fixed effect in each iteration by using the within-groups estimator. This amounts to replacing the linear regression in step 4) of the algorithm by a regression with deviations from the means of the second group. Second, in each iteration we retain the last two estimates of $\boldsymbol{\alpha}$ and use the three data points to adjust the trajectory of the estimates for the fixed effects. The estimation routines for the OLS coefficients, standard errors, and fixed effects are implemented in Stata and discussed in Guimarães and Portugal (2009).

Estimation of the standard error associated with the cycle variable Estimation of the standard error associated with the variable $cycle_t$ poses an additional problem because this variable lacks cross-sectional variation. A common solution is to calculate a standard error that is robust to (annual) clustering. However, implementation of the typical formula for clustered standard errors is problematic because it requires the inversion of the high-dimensional matrix of regressors. To circumvent this problem we compute the clustered standard errors in a final regression that treats the estimated fixed effects as an offset. We also implement an alternative approach suggested to us by Manuel Arellano. The approach consists of the following five-step procedure:

1) Obtain the OLS solution with worker and firm fixed effects and retain the estimated fixed effects;

2) Using the estimated fixed effects as an offset, run a regression of the wage variable on all regressors except the cycle variable. Keep the residuals from that regression;

3) Compute the yearly average of the residuals in 2) to obtain an aggregate series $\widehat{u}_t$;

4) Run a regression of the $cycle_t$ variable on all other regressors again using the estimated fixed effects as an offset. Call those residuals $\widehat{cycle}_t$.

5) Run a simple OLS regression of $\widehat{u}_t$ on $\widehat{cycle}_t$.

Both of these procedures provide a practical solution to estimate a credible standard error for the coefficient associated with the $cycle_t$ variable. However, a shortcoming of these procedures is that neither takes into account the fact that the fixed-effects had been previously estimated.