

From Lotka-Volterra systems to Polymatrix Replicators

Telmo Peixe^{1*}

(joint work with Hassan Alishah and Pedro Duarte)

¹ISEG - Lisbon School of Economics and Management, Universidade de Lisboa
and

Cemapre - Centro de Matemática Aplicada à Previsão e Decisão Económica

*telmop@iseg.ulisboa.pt

ABSTRACT

Consider a population divided in a finite number of groups, each one with a finite number of strategies, where interactions between individuals of any two groups are allowed, including the same group. This model is designated as the polymatrix game.

The system of differential equations associated to a polymatrix game, introduced recently by Alishah and Duarte in [3] and designated as *polymatrix replicator*, form a simple class of ordinary differential equations defined on prisms given by a product of simplexes.

This class of replicator dynamics contains well known classes of evolutionary game dynamics, such as the symmetric and asymmetric replicator equations, and some replicator equations for n -person games.

As J. Hofbauer proved in [2] the replicator equation is in some sense equivalent to the Lotka-Volterra (LV) system, independently introduced in 1920s by A. J. Lotka [1] and V. Volterra [6]. The LV system is perhaps the most widely known system used in scientific areas as diverse as physics, chemistry, biology, and economy.

In this talk we present the definition of the polymatrix replicator, some basic properties, and some results about the dynamics and the inferences we can make about the associated polymatrix game [4, 5].

References

- [1] Alfred J. Lotka (1958) *Elements of mathematical biology. (formerly published under the title Elements of Physical Biology)*, Dover Publications, Inc., New York.
- [2] Joseph Hofbauer (1981) *On the occurrence of limit cycles in the Volterra-Lotka equation*, Nonlinear Anal., Volume (5) , no. 9, 1003-1007.
- [3] Hassan Najafi Alishah and Pedro Duarte (2015) *Hamiltonian evolutionary games*, Journal of Dynamics and Games, Volume (2) , no. 1, 33-49.
- [4] Hassan Najafi Alishah, Pedro Duarte and Telmo Peixe (2015) *Conservative and Dissipative Polymatrix Replicators*, Journal of Dynamics and Games, Volume (2) , no. 2, 157-185.
- [5] Telmo Peixe (2018) *Permanence in Polymatrix Replicators*, submitted.
- [6] Vito Volterra (1990) *Leçons sur la théorie mathématique de la lutte pour la vie. (Reprint of the 1931 original)*, Les Grands Classiques Gauthier-Villars, Éditions Jacques Gabay, Sceaux.