ISEG - Lisbon School of Economics and Management

2[nd] Semester of 2019/2020

Continuous Evaluation - Exercise 7

1. Let X be a random variable such that its probability function is given by .

x	0	1	2	otherwise
$P(X = x)$	0.2	0.5	0.3	0

a) Compute the cumulative distribution function.
Solution: By definition, we can define the CDF doing

$$F_X(x) = P(X \leq x) = \begin{cases} 0, & x < 0 \\ 0.2, & 0 \leq x < 1 \\ 0.7, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$$

b) Compute the moment generating function.
Solution: The moment generating function is, by definition, given by

$$M_X(t) = E(e^{tX}) = \sum_{x=0}^{2} e^{tx} P(X = x) = 0.2 + 0.5e^t + 0.3e^{2t}$$

c) Compute the coefficient of variation.
Answer: The coefficient of variation of X is, by definition $\rho_X = \sigma_X / \mu_X$. Then, one has to compute

$$E(X) = M_X'(0) = (0.5e^t + 0.6e^{2t})|_{t=0} = 1.1$$
$$E(X^2) = M_X''(0) = (0.5e^t + 1.2e^{2t})|_{t=0} = 1.7.$$

Then, $Var(X) = E(X^2) - (E(X))^2 = 49/100$ and, consequently, $\sigma_X = \sqrt{49/100} = 7/10$. The coefficient of variation is

$$\rho_X = \frac{\sigma_X}{\mu_X} = \frac{7/10}{11/10} = \frac{7}{11}.$$

d) Compute the mode and the median.
Solution: The mode of X is given by

$$mo(X) = \arg\max_{x \in D_X} : P(X = x) = 1.$$

The median of X is given by

$$me(X) = \min\{x \in \mathbb{R} : F_X(x) \geq 0.5\} = \min\{x \in \mathbb{R} : x \geq 1\} = 1.$$

e) Let Y be the random variable given by $Y = 2X + 3$. Compute $M_Y(t)$.
Solution: By using the properties of the moment generating functions, we get that

$$\begin{aligned} M_Y(t) &= E\left(e^{tY}\right) = E\left(e^{t(2X+3)}\right) = e^{3t} E\left(e^{(2t)X}\right) \\ &= e^{3t}(0.2 + 0.5e^{2t} + 0.3e^{4t}) = 0.2e^{3t} + 0.5e^{5t} + 0.3e^{7t}. \end{aligned}$$