LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

MASTER

MONETARY AND FINANCIAL ECONOMICS

MASTER´S FINAL WORK

DISSERTATION

DOES LEVERAGE AFFECT ASSET RETURNS? THEORY AND EVIDENCE

PAULO GUILHERME RODRIGUES FILIPE RELVAS CORREIA

NOVEMBER - 2020

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SUPERVISION:

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Glossary

CDO Collateralised Debt Obligations. ii, 8

CDS Credit Default Swap. ii, 8, 18

DTI Debt-to-Income. ii, 4

ECB European Central Bank. ii, iii, vii, 1, 4, 24, 29, 43

GDP Gross Domestic Product. ii, vi, 1, 24–26

JEL Journal of Economic Literature. ii, iii

LTV Loan-to-Value. ii, iii, 4, 22, 25, 29

Abstract, Keywords, and Journal of Economic Literature (JEL) Codes

The housing bubble bust following the emergence of leverage-fuelled property booms observed in the US culminated in the financial crisis of 2008-09 did regain the interest on the vulnerabilities that result from higher levels of leverage. To capture the relation between leverage and its influence on asset bubbles, this paper presents a model to replicate an economy with two periods, two different assets, with one more risky than the other, where two states of nature can occur. The heterogeneous beliefs of agents belonging to this economy result in different assignments on the expected return of the risky asset which in turn dictates that only the more optimistic agents participate in the risky asset market. When agents have access to credit to purchase the risky asset, this participation rate diminishes as a result of the increasing price of the risky asset. One can also derive from this model that the rate of return of the risky asset depends positively on the agent's leverage ratio (or Loan-to-Value (LTV)). By using quarterly housing data from the European Central Bank (ECB) Data Warehouse, this paper does a multivariate regression analysis to find the housing return rate in the Euro area reacts negatively to increasing households' leverage observed in the current quarter, but has a higher positive reaction from the leverage level observed in the previous quarter. The results suggest the rate of return of the housing market may show some delay to reply to looser margin requirements.

Keywords: Credit; Leverage; Asset prices; House prices; Asset returns.

JEL Codes: C22; C68; G12; G21; G51.

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Does Leverage Affect Asset Returns? Theory and Evidence

By Paulo R. Correia

This dissertation intends to present a model based on the leverage cycle theory from Geanakoplos (2010) to capture the relation between debt and asset prices. The importance of this theory comes from the fact that it gives a very reasonable explanation for the mechanism that triggered the financial crisis of 2008-09. This study also seizes quarterly ECB data to present a multivariate regression analysis that suggests households' level of indebtedness has opposite effects on rates of return of the housing market in the Euro depending on the quarterly lag of the leverage ratio.

1 Introduction

In the last decades, we have seen advanced economies become more and more financialised, especially since the 1970s. Before that, there was a relatively stable relationship between bank lending to the non-financial private sector and the GDP, despite the deep contraction in bank lending between the Great Depression and World War II. However, there is an astounding surge of bank credit since the 1960s. In almost 30 years, the average bank credit-to-GDP ratio in advanced economies almost doubled from 63% in 1980 to 118% in 2009 as Figure 1 shows.[1]

The main driver of this surge in credit comes mostly from the increase in home-ownership rates as Jordà et al. (2016) suggest. The home-ownership rates of advanced economies were around 40% after World War II and then risen, during the past half-century, to 60% in the 2000s. This was only possible with access to credit that was not available before. As Figure 2 suggests, mortgage lending accounts for the lion's share of the rise of credit-to-GDP ratios in advanced countries since the 1990s. In 2016, the average mortgage lending already represented more than 65% of the GDP in the 17 advanced economies considered in Jordà et al. (2017)'s data-set. This fact resulted in the surge of global house prices that tripled within three decades during the financial liberalization period (Knoll et al., 2017).

The fact that the financial crisis of 2008-09 initiated in the U.S. in 2008-2009 was preceded by the emergence of leverage-fuelled property booms reminded us that financial factors play a crucial role in economic cycles bringing back theory focusing on credit cycles, which argues that credit build-up leads to economic expansion and credit tightening causes recessions, and if it persists, depressions. The pioneer in this literature was

[1] This increase should have been even higher since the data excludes the credit creation in the shadow banking system that has a considerable size in USA as argued in Pozsar et al. (2010).

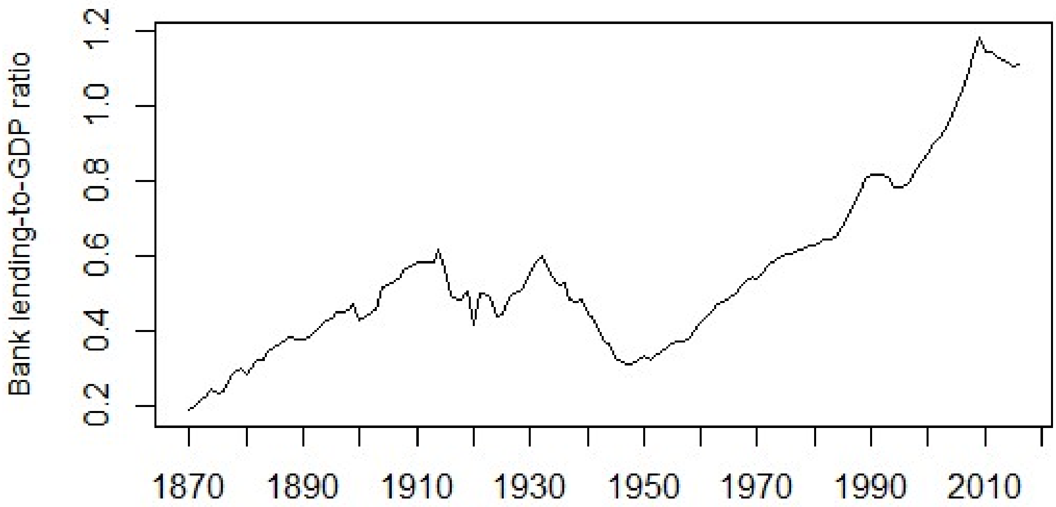


Figure 1: Average ratio of total bank lending to GDP

Note: The average ratio of total loans to GDP is calculated using the data provided by Jordà et al. (2017). This data set covers 17 advanced economies (Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, UK and USA).

Irving Fisher who argued that over-indebtedness can result in deflation in future periods which in turn can cause the liquidation of collateralised debt. His theory brought financial intermediation under the spotlight. According to Fisher (1933), the Great Depression was a result of a vicious cycle where the real burden was increased by the falling prices, which then led to further deflation.

Despite the central role of finance in business cycles assigned by Fisher, there is some influential literature that neglects the role of finance on the economy. A case of that is Modigliani and Miller (1958) who suggest that companies' economic decisions are independent of their financial structure. Other literature that was gaining interest prior to the last financial crisis was related to the real business-cycle theory which did not include financial frictions in their models (see, for example, Kydland and Prescott, 1982; Lucas Jr, 1977). However, since the burst of the largest leveraged boom in the history of the western economies, the notion that financial factors influence the real economy seems to regain interest in macro-economic thinking.

In the aftermath of the Great Recession of 2008-09, we have seen an emergence of new literature that reserve a central role for financial intermediation, by introducing financial variables in their models based on the general equilibrium theory. An example of that is the leverage cycle theory from an economist called John Geanakoplos from the

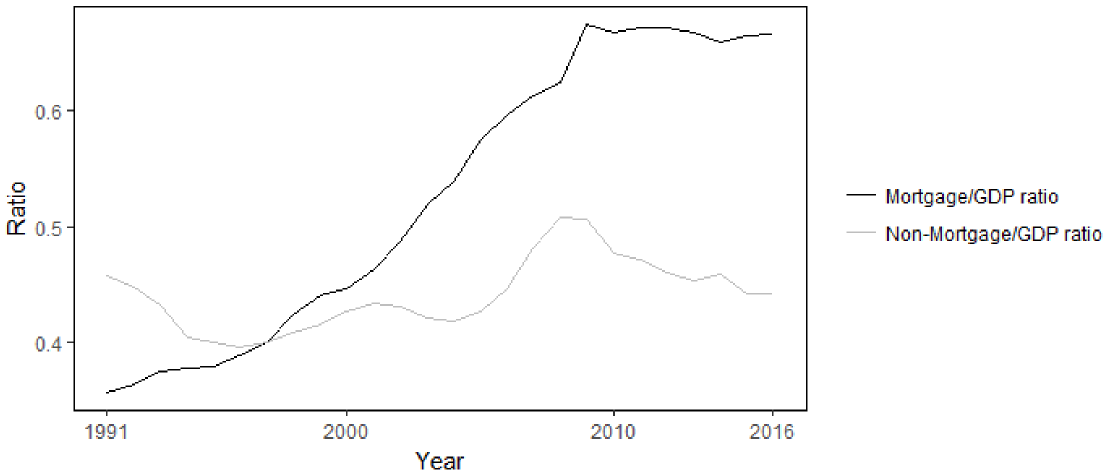


Figure 2: Average ratios of mortgage and non-mortgage lending to GDP

Note: These average ratios are calculated using the data provided by Jordà et al. (2017). This data set covers 17 advanced economies (Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, UK and USA).

University of Yale who provides a model to capture how collateral requirements affect asset prices and economic activity.

Geanakoplos (2010) argues that long periods of low volatility, such as the Great Moderation, together with financial innovation lead to increased leverage and looser credit standards. These phenomena raise asset prices and increase economic activity, but makes the economy more vulnerable. When bad news hits the horizon, we witness an increase in uncertainty that decreases asset valuation and tightens credit requirements. This tightening works as a feedback loop by further decreasing asset prices as well as economic activity. To make matters worse, financial innovation contributes to the emergence of new derivatives as Credit Default Swaps, which further depress prices until they become too low relative to debts, thus leading to default. According to Geanakoplos, this amplification effect of asset prices caused by leverage was the main trigger of the 2008 financial crisis.

The leverage cycle theory contrasts with the mainstream financial economic theory that focuses on interest rate behavior. Even if we assume constant interest rates, financial intermediation can contribute to increasing demand for assets, and consequently their prices, by allowing agents to leverage more, i.e., by allowing agents to pay down less and get their hands on more money to buy a given asset.

To better understand what is leverage, imagine if we buy a durable good, for instance, a house that costs €100 and we pay down €40 with cash, one can say the margin requirement on this purchase equals 40%. Knowing the rest of the money used in the purchase is borrowed money, one can also state the leverage on this purchase, given by the LTV ratio, is 60%. A few years later, imagine we can buy another house of €100 but the leverage ratio associated is 90%. Now the buyer only needs €10. This looser margin requirement should imply a higher demand for houses and then a surge in house prices. Prior to the crisis of 2008, the US home buyers could get a mortgage loan with only a 3% margin requirement. This caused house prices to soar (Geanakoplos, 2010).

In the resolution of that crisis, we observed that supervisors begin to be more attentive to purchases on leverage, such that nowadays, the requirements for buying assets like houses are much more tighter than were before the crises. That fact proves the relevance of regulation on collateral requirements, like limiting LTV and Debt-to-Income (DTI) ratios, to reduce over-borrowing, and increase financial stability.

This dissertation intends to simplify the model presented by Geanakoplos in his several studies on leverage and assess the influence of the level of indebtedness on house prices with help of the ECB data to examine how house return rates in the Euro area react to households' leverage ratio, which is measured by dividing Euro area households' outstanding debt to their wealth.

To replicate Geanakoplos' theory, a baseline model is first presented in which there are no debt contracts available. Agents can only buy risk-free or risky assets by selling the endowment of the asset they don't want. In the second model, agents already have access to the credit market by issuing non-contingent promises, that represent loans, using assets as collateral. The collateral is owned by the borrower but it may be confiscated by the lender (more concretely by the court on behalf of the lender) if the borrower does not deliver his promises. Nevertheless, as we will see further, there is no default in equilibrium since the collateral requirements will be set high enough to rule out default. This idea follows the Binomial No-Default Theorem presented by Fostel and Geanakoplos (2015) who show that equilibrium in an economy with financial assets serving as collateral is the same regardless of the possibility of default. What affects the equilibrium is the potential default but not the default itself, such that in equilibrium, borrowers will promise as much as they can while assuring their lenders the collateral is enough to guarantee delivery. In the case that only non-contingent debt contracts are allowed in the economy as presented in this paper's model (as it typically occurs in most of the loan contracts), agents will end up choosing to trade contracts where the bad outcome is totally secured by the collateral.

The dissertation is organized as follows: the second section briefly revises the existing literature about this topic; section 3 introduces the model which captures the asset price fluctuations implied on the rise of leverage; the fourth section presents a multivariate regression analysis to assess the relationship between households' leverage and housing returns in the Euro area; finally, the conclusion is summed up in section 5.

2 Literature Review

Despite the subprime crisis being triggered by a leveraged-boom in the American housing market, there are some economic theories that do not consider finance and debt as relevant variables. A clear example of that is the influential Modigliani-Miller Theorem which argues the value of a firm does not depend on the funding source (Modigliani and Miller, 1958). The theorem defends that, even if all the firm's assets were financed through debt, the value of the firm would be the same as if the capital structure was solely composed of equity capital. However, it is noteworthy that the MM theorem only considers this idea valid in a world without taxes. In a world with taxes (the real world), when the interest on the debt is tax-deductible, the theorem indicates that there is a directly proportional relationship between the increase in the value of the company and the amount of debt used.

Another theory that neglects the role of finance is the real-business cycle theory. In a very short version, this theory envisioned that technological shocks that provoke random fluctuations in the productivity level are what make the constant output growth trend up or down. In this theory, finance has no role in economic cycles (Kydland and Prescott, 1982, is one of the most influential in this theory).

Despite the influence that these theories continue to have on economic thinking, there are empirical studies suggesting the growing relevance of debt in advanced economies. Schularick and Taylor (2012) and Jordà et al. (2016) found that over the second half of the twentieth century, there has been an unprecedented surge in the ratio of aggregate private credit to income in advanced economies which calls for the relevance of finance in economic theory. More recently, Jordà et al. (2017) showed that besides higher leverage going hand in hand with less volatility, it implied more severe tail events, meaning that expansion of private credit seems to be safe for small shocks, but dangerous for big shocks. In other words, leverage exposed advanced economies to bigger rare-event crashes, but it helped smooth small disturbances. This article also observed the great leveraging of the second half of the twentieth century took place primarily in the household and not in the corporate sector. Before, the same authors already reminded for the critical role of finance in the economy by showing that what makes a bubble dangerous is credit (Jordà et al., 2013, 2015). In a leveraged bubble, a positive feedback loop, resulting in a growing credit level and asset prices, is developed. The bust of this bubble results in deleveraging which weakens business and household spending, therefore debilitating economic activity altogether and augmenting macroeconomic risk in credit markets. In the same way, Mian and Sufi (2015) support the idea that economic disasters like the Great Recession of 2008 are always preceded by growing household debt.

In the wake of the Great Depression of 1929 that shook the world economy, Irving Fisher developed the debt-deflation theory that culminated in the publication of Fisher (1933). This theory was a pioneer in bringing credit to a central role in economic thinking by attributing the crises to the bursting of a credit bubble. In turn, this bubble burst unleashes a series of negative effects that harm the real economy such as the contraction of the money supply as bank loans are paid off, the fall in the level of asset prices, and the reduction in firms' profits and output. Fisher argued that growing real debt burdens are a result of strong declines in nominal incomes and the price level, which in turn leads to default. As a result, the aggregate demand is lowered and the price level declines further, thus leading to a debt-deflation spiral. When there is a severe deflation, debtor bankruptcies along with falling asset prices lead to a decline in the nominal value of assets on bank balance sheets. This will cause the banks to tighten their margin requirements, which in turn lowers consumption and investment leading to a decline in the aggregate demand, thereby harming the economy by further contributing to the deflationary spiral.

Following Fisher's work, Minsky (1986) presented a Financial Instability Hypothesis. According to this hypothesis, financial institutions tend to invest more in riskier assets when there are prolonged periods of prosperity and optimism about future prospects, which in turn can make the economic system more vulnerable in the case that default materializes. Later, Keen (1995) modelled Minsky's Financial Instability Hypothesis.

Bhattacharya et al. (2011) support Minsky's work by examining the interaction between the financial stability and leverage cycle. The paper presents a model that shows the financial institutions becoming more optimistic about the future results in increasing leverage and shifting their portfolios towards riskier projects to pursue higher returns. Thus, when bad news realises, the consequences for financial stability are more severe and the default is higher as a result of this abrupt shift in the financial institutions' expectations.

Still, on financial frictions, Adrian and Boyarchenko (2012) show that lower consumption volatility and higher consumption growth are generated by higher leverage in normal times at the expense of endogenous systemic financial risk. Similarly, Brunnermeier and Sannikov (2014) present a macro-financial model that shows how the financial system becomes less stable by leverage, thereby leading to increasing systemic risks. Other papers that capture the deflationary effects à la Fisher leading to financial instability include Shleifer and Vishny (1992), Kiyotaki and Moore (1997), Kyle and Xiong (2001) and Morris and Shin (2004).

Following all this influential literature that I have mentioned so far, which demonstrates the growing relevance of financial factors in economic thinking, comes the theory of leverage developed by Geanakoplos on which the model presented in section 3 is based. This theory was a pioneer in the general equilibrium analysis of collateralised lending to explain an alternative mechanism of how credit affects asset prices and business cycles. Firstly, Geanakoplos (1997) showed how incomplete markets lead agents to leverage as a consequence of the heterogeneous beliefs of economic agents which creates large price fluctuations. Later, Geanakoplos (2003) showed there is a liquidity crisis when there is bad news about an asset that raises its probability of default. Moreover, the article showed the crisis is amplified by the higher collateral requirements that come from that increase in the probability of default. The paper further argues these crises can easily spill over to several markets. Following, Fostel and Geanakoplos (2012) presented a model that tried to demonstrate the effect of financial innovation, like leverage and tranching (the latter became popular with the appearance of the Collateralised Debt Obligations (CDO) when the last financial crisis was brewing), in amplifying the fluctuations in asset prices that resulted in the subprime crisis. Fostel and Geanakoplos (2016) proved that a collateral constrained agent, who is given the opportunity to borrow money without posting collateral, will never go for investment projects (the ones that usually demands collateral) and instead spend the money on consumption.

Summing up, one can say the model of collateral equilibrium developed by the economist John Geanakoplos, started in Geanakoplos (1997, 2003), showed how heterogeneous beliefs of economic agents lead to leverage which in turn raise asset prices. Then, Geanakoplos (2010) introduced short-selling in the same kind of model to replicate the appearance of Credit Default Swap (CDS), which lowers the asset prices and amplifies economic recessions. Afterward, Fostel and Geanakoplos (2012) developed a more general model that showed how different kinds of financial innovations like leverage, tranching, and CDS can have big effects on prices. Finally, Fostel and Geanakoplos (2016) introduced investment in the model to show that investment is also affected by financial innovation.

Similarly to Geanakoplos (2003), Eggertsson and Krugman (2012) set a flexible-price endowment model in which "impatient" agents borrow from "patient" agents subject to a debt limit. If there is a shock that reduces the debt constraint, a vicious deleveraging spiral is initiated as a result of agents forced to cut spending. In the same way, Simsek (2013) presents a model in which traders borrow by selling collateralised contracts to lenders who do not share the same beliefs. However, since collateral is not as much valued by the pessimists as compared to optimists, they are reluctant to lend, which puts an endogenous constraint on the ability of the optimists to borrow and to influence the prices of asset. Previously, Reinhart and Rogoff (2008) had already identified that a potential cause for the increase in asset prices leading to the housing and mortgage crisis was the optimism of a fraction of investors. Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009) are other papers that present some more insights on the interaction between the borrowing capacity of agents and the pricing of assets.

It is also noteworthy the Geanakoplos' literature on leverage contrasts with the several articles that focus their models on asymmetric information to show how the collateral ratio is determined in equilibrium. For example, based on the strong informational asymmetries, Bernanke and Gertler (1989) modelled a collateral-driven credit constraint, whereby fully collateralised loans can only be obtained by the firm if the value of the assets of the firm is greater than the value of the loan. In this paper, deflationary pressures on the prices of the firm's assets shrink the available amount of credit for the firm. Subsequently, Bernanke and Gertler (1990) developed a model of the process of investment finance in which there is asymmetric information between lenders and borrowers about the borrower's effort and about the quality of investment projects, such that the number of projects initiated depends on the balance sheets of the borrower. In the case of weak balance sheets, the economy experiences misallocation of investment resources or underinvestment. Townsend (1979), Myers and Majluf (1984), Gale and Hellwig (1985) and Hart and Moore (1994) are other works that focus on asymmetric information as the main friction in the credit markets.

In the next section, a simpler version of the collateral equilibrium model has been introduced.

3 A Simple Model

As already mentioned, Geanakoplos (1997, 2003, 2010) and Fostel and Geanakoplos (2012, 2015, 2016) describe a model of *collateral equilibrium*. The basic idea is the following: In the financial market, two types of agents participate, optimists (or investors) and pessimists (general public). Optimists invest in the risky asset M and pessimists invest in the risk-free asset F. If those agents have access to credit, not only assets are traded but also debt contracts in such a way that optimists sell promises to pessimists, which means that optimists become borrowers and pessimists become lenders.

In equilibrium, four variables are determined: the number of assets each type of agent chooses to hold, the price of the risky asset, the quantity of promises each type of agent chooses to sell/buy, and the price of the promise which is equivalent to the underlying amount of each loan (evidently the latter two are only determined in the case with borrowing).

To simplify, this model only considers financial assets as equity stocks, which is an asset that does not give direct utility to their holders and pays the same dividends no matter who owns it. We will also assume this type of asset not carrying asymmetric information problems like a moral hazard or adverse selection. We will consider that the borrower has no control or specialized knowledge of the cash flows provided by the collateral.

3.1 Baseline Model

To introduce, a two-period model is presented in which borrowing is not allowed. This model is based on the one produced in section II.A of Geanakoplos (2010), though different notation has been used along with some simplification.

Figure 3 presents a binomial information tree with the two states that can occur in the second period, U and D. The good state is assumed to have as probability π_U (represents the probability of good news) and the bad state $\pi_D = 1 - \pi_U$ (probability of bad news). These probabilities are subjective.

Agents in this economy are indexed by a in a continuum between 0 and 1, $a \in [0, 1]$ ranked from more pessimistic to more optimistic as shown in Figure 4, which means that agent's beliefs are heterogeneous. This type of distribution of agents represented by a continuum allows different assignments to the probabilities of the states of nature for each agent a, depending on their optimism. Nevertheless, these agents are assumed to be risk-neutral and characterized by a linear utility function for consumption such that

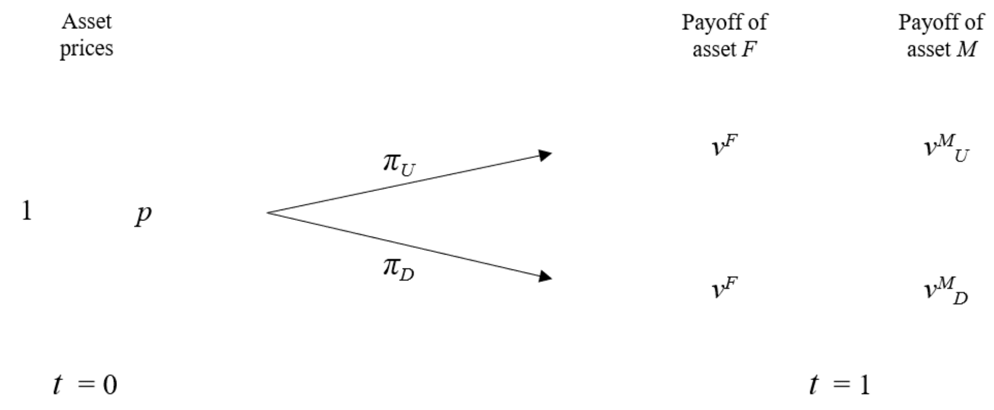


Figure 3: Binomial Information Tree

the von-Neumann-Morgenstern expected utility of agent a is

$$U^a(c_U, c_D) = \pi_U^a c_U + \pi_D^a c_D \tag{1}$$

Agents of this economy get no utility from holding the assets F and M and there is only one consumption good with price equal to one, for simplicity, which is only consumed in the future, at time $t = 1$. This consumption good is obtained out of the financial income depending on the holdings of agent a.

The model also assumes that agents are not impatient (i.e., they have a discount factor equal to 1). Agents are homogeneous in the other dimensions, including in their initial distribution of wealth, which means the initial endowments of assets F and M are equal for every agent $a \in [0, 1]$. Thus, in the current period, at time $t = 0$, agent a has an endowment of the two assets, a risk-free asset F and a risky asset M, with asset prices at time $t = 0$ and payoffs at time $t = 1$ denoted respectively by

$$\text{endowments} = (e^F, e^M),\ \text{asset prices} = (1, p),\ \text{payoffs} = \begin{pmatrix} v^F & v_U^M \\ v^F & v_D^M \end{pmatrix},$$

or equivalently the asset returns

$$(R^F, R^M) = \begin{pmatrix} v^F & \dfrac{v_U^M}{p} \\ v^F & \dfrac{v_D^M}{p} \end{pmatrix}.$$

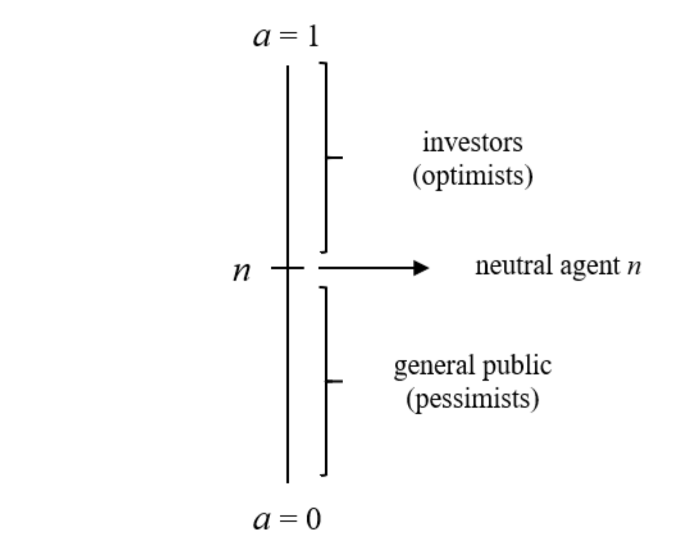


Figure 4: Continuum of Agents

We can think of the risk-free asset F as money or a term deposit, and M as an equity stock that may have a higher or a lower value in the future depending on the state that occurs.

Given asset prices and potential payoffs, each agent a chooses asset holdings f of F and m of M at time $t = 0$ and the consumption plan for time $t = 1$ in order to maximize its utility function (1), subject to the following sequence of budget constraints:

$$\begin{aligned} B^a(p) = \{(f, m, c_U, c_D) \in \mathbb{R}^4_+ :& \\ &(f - e^F) + p(m - e^M) = 0, \\ &c_s = fv^F + mv^M_s,\ s = U, D\} \end{aligned}$$

3.1.1 General equilibrium

Definition: Equilibrium in this economy is defined by $\left(p, \left(f, m, \{c_s\}_{s\in\{U,D\}}\right)_{a\in[0,1]}\right)$ such that

- agents solve their problems, i.e. $(f^a, m^a, c^a_U, c^a_D) \in$ argmax$\{U^a(c_U, c_D) : (f, m, c_U, c_D) \in B^a(p)\}$ for every $a \in [0, 1]$
- asset markets clear to guarantee the supply of assets F and M equals its demand

in this economy such that

$$\int_0^1 f^a da = e^F \tag{2}$$

$$\int_0^1 m^a da = e^M \tag{3}$$

3.1.2 General public, investors and the neutral buyer

Solving the agent problem described above[2], we get that

- Agent a is a member of the "general public" (those who are pessimists and only invest in the risk-free asset F) if and only if

$$v^F > \frac{\pi_U^a v_U^M + \pi_D^a v_D^M}{p} \tag{4}$$

  which means that according to the probability they assign to each state of nature, the general public believes the return of the risk-less asset is higher than the return of the risky asset.

- Agent a is a risky asset M "investor" (or an optimist) if and only if

$$v^F < \frac{\pi_U^a v_U^M + \pi_D^a v_D^M}{p} \tag{5}$$

  which means an investor believes that the return of the risky asset is higher than the return of the risk-less asset.

- Agent a is the neutral buyer $n \in (0, 1)$ that is indifferent between investing in the risk-less and the risky asset because he/she believes the return of both assets are equivalent. Considering the two previous conditions, (4) and (5), by continuity we get

$$v^F = \frac{\pi_U^n v_U^M + \pi_D^n v_D^M}{p} \tag{6}$$

  where π_s^n for $s = U, D$ are the probabilities associated with the neutral buyer.

Note the previous conditions make sense since the members of the general public attribute a lower probability to the state U than an investor does when it is this good state that provides a bigger payoff from holding the risky asset.

[2] You can see how to solve it in Appendix A.1.

3.1.3 Equilibrium conditions

Because agents are distributed in the domain $[0,1]$ and we know the arbitrage condition of the "frontier agent", we have to split the interval in some way. If we assume that "investors" assign a higher probability to the good state, we need to separate the population in the following way: if $a \in [0,n)$, then a is a member of the "general public"; if $a \in (n,1]$, he is an "investor".

Consequently, using this distribution of the population and the budget constraints, one can write the demand of agent a for the assets

$$f^a = \begin{cases} e^F + pe^M, & a \in [0,n) \\ 0, & a \in (n,1] \end{cases} \tag{7}$$

$$m^a = \begin{cases} 0, & a \in [0,n) \\ \dfrac{e^F + pe^M}{p}, & a \in (n,1] \end{cases} \tag{8}$$

Then, if we run the asset market clearing conditions (2) and (3), we obtain the equilibrium price of the risky asset M as

$$p = \left(\frac{1-n}{n}\right)\epsilon \tag{9}$$

where ϵ is the relative supply of the risk-free asset relative to the risky asset

$$\epsilon \equiv \frac{e^F}{e^M}.$$

Now we know the equilibrium price of M depends positively on the participation rate in the risky asset market given by $1-n$. The variable n which is determined endogenously, gives the population share of the general public, meaning that the lower the share of pessimists in this economy, the higher the price of the risky asset. Note that in equilibrium, the price of the risky asset equals the amount of risk-free assets sold by investors divided by the amount of risky assets they bought from the general public.

Now, one can write the expressions for the equilibrium in asset holdings and consumption plan through the market clearing conditions

$$f^a = \begin{cases} \dfrac{e^F}{n}, & a \in [0,n) \\ 0, & a \in (n,1] \end{cases} \tag{10}$$

$$m^a = \begin{cases} 0, & a \in [0, n) \\ \dfrac{e^M}{1-n}, & a \in (n, 1] \end{cases} \tag{11}$$

and

$$c_s^a = \begin{cases} \dfrac{e^F v^F}{n}, & a \in [0, n) \\ \dfrac{e^M v_s^M}{1-n}, & a \in (n, 1],\ s = U, D \end{cases} \tag{12}$$

Observe that all future consumption is derived from asset payoffs and it is state-independent for the "general public" while it is state-dependent for the "investors".

3.1.4 The endogenous participation rate

The equilibrium values for p, f and m depend on the endogenous split of the population n, or endogenous participation in the asset markets.

In order to be able to determine n, the idiosyncratic probabilities of agents are assumed to be a function of a, i.e., $\pi_s^a = \pi_s(a)$, for $s = U, D$, such that the probability of good news (good state) for neutral buyer n is $\pi_U^n = n$ and the probability of bad news is $\pi_D^n = 1 - n$, as it is presented in Geanakoplos (2010).[3]

Using this assumption and substituting the equilibrium asset price (9) in equation (6), we get the equilibrium population split

$$n^* = n(\epsilon, \delta_U, \delta_D) = -\frac{\epsilon + \delta_D - \sqrt{(\delta_D - \epsilon)^2 + 4\,\epsilon\,\delta_U}}{2\,(\delta_U - \delta_D)} \tag{13}$$

with δ being the risk premium in multiplicative form

$$\delta \equiv (\delta_U, \delta_D) = \left(\frac{v_U^M}{v^F}, \frac{v_D^M}{v^F} \right)$$

where $\delta_U > \delta_D$.

Note that besides giving the equilibrium neutral buyer, n^* also gives us the equilibrium percentage of pessimists in this economy, as $1 - n^*$ gives the equilibrium participation rate in the risky asset market. This equilibrium participation rate is a function of the risk premium and relative supply of assets.

[3]In Fostel and Geanakoplos (2012), it is assumed that $\pi_U(a) = 1 - (1-a)^2$, whereas Fostel et al. (2017) assumes that $\pi_U(a) = a^\zeta$ with $\zeta < 1$ where ζ determines the relative proportion of optimists vs pessimists.

To analyse the relationship between the participation rate and their explanatory variables, partial derivatives of n were taken with respect to those variables.

- $\frac{\partial n}{\partial \delta_U} < 0$, meaning the bigger the risk premium when the good state occurs, the bigger the share of investors, $1 - n$, participating in the risky asset market in this economy.
- $\frac{\partial n}{\partial \delta_D} < 0$, which means that an increase in the risk premium for the bad state also results in an increase of the share of optimistic agents in the economy. Thus, regardless of the state of nature that occurs, one can conclude there is always higher participation in the risky asset market as risk premium increases.
- $\frac{\partial n}{\partial \epsilon} > 0$, showing that when the supply of risk-free assets increases relatively to the supply of risky assets, the participation rate in the risky asset market diminishes, and we tend to have more pessimists to hold these risk-less assets.

3.1.5 Numerical Example

Table I: Exogenous variables

	e	v_U	v_D
F	1	0.5	0.5
M	1	1	0.2

Now, the numerical example presented in Table I will be used for the exogenous variables e^F, e^M, δ^F and δ^M_s for $s = U, D$

From this data and using equation (13), we get $n \approx 0.466$, meaning that investors, i.e. every agent $a > 0.466$, will sell all their endowment of the risk-free asset F to buy all they can of risky asset M and the bottom 53.4% will demand the risk-free assets the top 46.6% are selling. Then substituting this value in equation (9), we get the equilibrium price for the risky asset $p \approx 1.146$. Finally, the equilibrium distribution of asset holdings and consumption can be obtained from equations (10), (11) and (12).

Table II: Equilibrium portfolios and consumption for an economy with no access to credit

	f	m	c_U	c_D
$a \in [0, n)$	2.146	0	1.073	1.073
$a \in (n, 1]$	0	1.873	1.873	0.375

Further, these results will be compared with the results given by the model where there is access to the credit market.

3.2 A Model with access to the Credit Market

This section presents a model with similar features than the previous one, that is, replicating an economy with two periods, two different assets, with one more risky than the other, where two states of nature can occur and agents are risk-neutral and have heterogeneous beliefs, being homogeneous in the other dimensions. The only difference is that in this economy, borrowing is allowed to give the most optimistic agents a chance to spend more on the risky asset. This will push up the price of M as we will see further.

To introduce borrowing,the agent a is supposed to only make non-contingent promises backed by the asset M such that each debt contract d uses one unit of asset M and promises (d_s, v_s^M) units of consumption, whatever it is the state of nature that occurs. In other words, we define a debt contract as an ordered pair consisting of a promise and the collateral backing it.

The price of contract d is given by θ_d and it is determined in the first period, meaning that agent a can borrow θ_d today by selling the debt contract d promising d tomorrow.

To enforce the repayment of a loan, the investor uses asset M as the collateral so that if he defaults, the asset can be seized. Typically, default happens when the collateral worth less than the promise. However, since we assume loans are non-recourse, the maximum borrowers can lose is their collateral if they do not honor their promise, even if the value of the collateral is lower than the promise.

Now let ψ^a be the number of contracts d traded by an agent a at time $t = 0$. A positive ψ^a indicates agent a is selling contracts d and borrowing $\psi^a\theta_d$. A negative ψ^a indicates agent a is buying contracts d, that means, he/she is lending $|\psi^a|\,\theta_d$. The agent who sells promises invests in the risky asset and offers the payoff $\min\{d_s, v_s^M\}$ to the lender, where, given the loans being non-contingent, we assume the contracted d is state-independent such that $d_U = d_D = d$.

Given the asset and debt contract prices (p, θ_d), each agent $a \in (0, 1)$ chooses risk-free and risky assets holdings, (f, m), and debt contract trades, ψ^a, at $t = 0$, and consumption, c_s, in each state s, at $t = 1$ to maximize utility function (1) subject to the budget set given by

$$\begin{aligned} B^a(p,\theta) = \big\{ &(f, m, c_U, c_D, \psi) \in \mathbb{R}^4_+ \times \mathbb{R} \; : \\ &(f - e^F) + p\,(m - e^M) = \theta\,\psi\;, \\ &max\{0, \psi\} \leq m\;, \\ &c_s = f\,v^F + m\,v_s^M - min\{d_s, v_s^M\}\,\psi,\;\; s = U, D \big\} \end{aligned}$$

Now the budget constraint allows asset holdings chosen by agent a at $t = 0$ to go beyond the revenue from his initial endowment by buying/selling debt contracts. Nevertheless, the number of debt contracts available to be sold by agent a is limited to the number of assets M he chooses to hold, such that agent a can not sell more promises than his holdings of asset M that he can use as collateral. The last constraint shows what will be the consumption level after the agent a has paid (in the case of agent a being an investor) or has received (agent a being a member of the general public) the value promised as it was settled in the debt contract.

In the upcoming section, the introduction of credit in form of promises will allow optimistic agents to buy all the risky assets M by leveraging their purchases selling the debt contract d, at price θ, using the asset M as collateral. In doing so, it is like optimists are buying the Arrow security that pays in the U state (since at D, their net payoff after debt repayment is 0).[4]

3.2.1 General equilibrium

Again, equilibrium in this economy is defined by $\Big(p, \theta, \big(f, m, \psi, \{c_s\}_{s\in\{U,D\}}\big)_{a\in[0,1]}\Big)$ such that agents solve their problems, i.e. $(f^a, m^a, \psi^a, c_U^a, c_D^a) \in \text{argmax}\{U^a(c_U, c_D) : (f, m, \psi, c_U, c_D) \in B^a(p, \theta)\}$ for every $a \in [0, 1]$ and asset markets clear, but now the market for promises also clears

$$\int_0^1 \psi^a da = 0 \tag{14}$$

This new market-clearing condition ensures that for each borrower i who sells a determined promise, there is a lender g who wants to buy it.

Since the loans are assumed as non-contingent, which implies that promises are state-independent such that $d_U = d_D = d$, in order to get an equilibrium, it makes sense to assume that a contract between the public and an investor can only be drawn if

$$\mathbb{E}^g[\min\{d, v^M\}] = \mathbb{E}^i[\min\{d, v^M\}]$$

This expression ensures that in equilibrium, the expected outcome from the promise contracted between an investor and the public is the same. By assuming this, we get that there is only equilibrium in the promises market if the payoff of the promise equals the collateral value if the worst scenario occurs, i.e., if d equals v_D^M. Hence, we can already state that in equilibrium, the collateral requirements will be set high enough to

[4] Geanakoplos (2010) and Fostel and Geanakoplos (2012) also show how agents can short-selling the asset M and effectively buy Arrow D security. These articles introduce a CDS on asset M that is a contract that promises to pay 0 when M pays v_U^M and promises $v_U^M - v_D^M$ when M pays v_D^M.

rule out default.[5]

3.2.2 General public, investors, and the neutral agent

Again, solving the agent problem, we get some results similar to the model without borrowing.[6] The general public still believes the return of asset F is higher than the expected return of asset M, as equation (4) shows, while investors think otherwise, as shown by equation (5) in the previous model. The neutral agent continues to be indifferent between investing in the risk-less and the risky asset because he believes the return of both assets are equivalent. Again this happens because the members of the general public attribute a lower probability to the state U than an investor does when it is this good state that provides a bigger payoff from holding the risky asset.

3.2.3 Equilibrium conditions

Now the demand of agent a for the risk-free asset and the risky asset become

$$f^a = \begin{cases} e^F + pe^M + \theta\psi^a, & a \in [0, b) \\ 0, & a \in (n, 1] \end{cases} \tag{15}$$

$$m^a = \begin{cases} 0, & a \in [0, n) \\ \dfrac{e^F + pe^M + \theta\psi^a}{p}, & a \in (n, 1] \end{cases} \tag{16}$$

In equilibrium, the investor $a \in (n, 1]$ chooses to borrow as much as he can such that the number of promises ψ^a equals his holdings of the risky asset m^a. This only holds when the investor expects that the return of the risky asset M will be higher than the return of the promises he sold to the general public. Hence, the equilibrium number of promises in this economy becomes

$$\psi^a = \begin{cases} -\dfrac{1-n}{n}\dfrac{e^F + p\,e^M}{p - \theta^*}, & a \in [0, n) \\ \dfrac{e^F + p\,e^M}{p - \theta^*}, & a \in (n, 1] \end{cases} \tag{17}$$

[5] It follows Geanakoplos (2003) which confirms the only debt contract traded in equilibrium is $d^* = v_D^M$.
[6] You can see how to solve it in Appendix A.2.

where

$$\theta^* = \frac{v_D^M}{v^F}$$

is the equilibrium price of promise d^* which gives the value borrowed by investors to buy each risky asset he chose to hold.

From the previous expression, it can be verified that the higher the price of the risky asset, the lower the number of promises each investor chooses to sell since there will be fewer agents able to buy risky assets. It can also be stated that the equilibrium price of a promise has a positive effect on the number of promises sold by investors since a higher θ^* provides more money to investors from selling promises such that it is likely they supply more of those promises.

Again, if one runs the asset and promises markets clearing conditions (2), (3) and (14) one obtains the equilibrium price of the risky asset M in an economy where borrowing at exogenous collateral rates is allowed

$$p = \frac{1-n}{n}\epsilon + \frac{v_D^M}{n\, v^F} \tag{18}$$

which is analog to equation (9) in the previous model. Note that again the equilibrium price of the risky asset equals the amount of money investors can get by selling their risk-free assets and by borrowing, divided by the amount of risky assets they can buy.

Through the market-clearing conditions, we get the same expressions for the equilibrium asset holdings as presented in equations (10) and (11) in the model without borrowing, unlike what happens with the consumption plan that becomes

$$c_s^a = \begin{cases} \dfrac{e^F v^F + e^M v_D^M}{n}, & a \in [0, n) \\ \dfrac{e^M \left(v_s^M - v_D^M\right)}{1-n}, & a \in (n, 1],\ s = U, D \end{cases} \tag{19}$$

Whereas the public's consumption plan is affected positively by the payoff in the bad state got from the risky asset M (remember that $d = v_D^M$ in equilibrium to meet the expectations of investors and the public regarding making contracts on loans), the investors' consumption plan is affected negatively by v_D^M since it's the value of the repayment from their loans. Yet, if the bad state occurs, investors will not consume anything at $t = 1$, and the consumption plan for the general public is still state independent besides the introduction of borrowing.

3.2.4 The endogenous participation rate

Again, since the probability of good news (good state) for neutral buyer n is $\pi_U^n = n$ and the probability of bad news is $\pi_D^n = 1 - n$, one can determine n^* substituting the equilibrium asset price (18) in equation (6) as follows:

$$n^* = n(\epsilon, \delta_U, \delta_D) = -\frac{\epsilon + \delta_D - \sqrt{(\delta_D + \epsilon)(4\delta_U - 3\delta_D + \epsilon)}}{2(\delta_U - \delta_D)} \tag{20}$$

The relation of the agents' participation rate in the risky assets' market regarding their explanatory variables is the same as presented in the model without borrowing, except for δ_D, such that an increase in the risk premium for the bad state results in an increase of the percentage of pessimistic agents in the economy (those who do not want to participate in the risky asset market). This result makes sense since the consumption plan for the general public now depends positively on the outcome of the contract they bought, which in turn depends on the payoff of the risky asset in the bad state v_D^M which is the collateral value they receive in case of default.

3.2.5 Numerical Example

Now, the same numerical example as in Table I will be used for the exogenous variables e^F, e^M, v^F and v_s^M for $s = U, D$.

From this data and using equation (20), we get $n \approx 0.595$, meaning the general public, i.e., every agent $a < 0.595$, will sell all their endowment of the risky asset and the top 40.5% (investors) will buy as many risky assets as they can by selling their endowment of risk-free assets and borrowing what they can from the bottom 59.5% (general public). Then substituting this value in equation (18), we get the equilibrium price for the risky asset $p \approx 1.352$. Finally, we can obtain the equilibrium distribution of asset holdings, the number of promises and consumption, from equations (10), (11), (17) and (19).

Table III: Equilibrium portfolios, number of promises and consumption for an economy with access to credit

	f	m	ψ	c_U	c_D
$a \in [0, n)$	1.681	0	-1.681	1.176	1.176
$a \in (n, 1]$	0	2.469	2.469	1.975	0

Comparing these results to the ones presented in Table II where there is no access to credit, we observe the price of the risky asset is higher in the economy with borrowing since there is a higher demand for the risky asset and agents choose to hold more of this type of asset. However, the participation rate on the risky asset market diminishes when borrowing is introduced, which means that only the more optimistic agents accept to hold the risky asset at a higher price. Yet the consumption level of the general public increases with the introduction of leverage since these agents are also lenders and they are paid back in the second period when it's time for their consumption. Investors also consume more when the good state occurs. However, if it does not occur, investors have nothing to consume unlike what happens in the economy without borrowing available.

3.2.6 Leverage, bad news and risky asset returns

Since we know the equilibrium price of the promise is 0.4, we can also get the leverage ratio, given by the LTV, by dividing the amount borrowed by the value of the asset used as collateral. So, in equilibrium, every debt contract d has an associated $leverage_d$ of 0.30 which means an investor i can resort to credit to purchase one asset M but needs to use 70% in cash.[7]

In order to relate the agent's leverage ratio to asset returns, one can derive the equilibrium risky asset return as follows

$$R^M = leverage_d \, \frac{n \, v_U^M + (1-n) \, v_D^M}{\delta_D} \tag{21}$$

Using the numerical example, we get $R^M = 0.5$, which is similar to the risky asset return in the economy without credit. The equation 21 now allows to state that the risky asset return depends positively on the agent's leverage ratio.

Remembering that in the beginning, the idiosyncratic probabilities of agents were assumed to be a function of a such that the probability of state D for the neutral agent n is given by $1 - n$, it can now be stated that the higher the probability of a bad outcome, the higher become the margin requirements (this is similar to stating that the lower the $\pi_U^n = n$, the lower the leverage ratio). This idea follows Geanakoplos (2003) which shows there is a liquidity crisis when there is bad news about an asset that raises its probability of default, but the crisis is amplified by the higher collateral requirements that come from that increase in the probability of default. That's why the margin requirements on debt contracts are so relevant to understand the dynamics between finance and economics.

Next, a multivariate regression analysis is presented to find how the housing return

[7] The leverage ratio associated to the debt contract d is obtained by dividing the debt contract by the price of its collateral, such that $leverage_d = \frac{\theta_d}{p}$. Similarly, the margin requirement associated to debt contract d is given by 1 - $leverage_d$.

rate reacts to the households' leverage ratio in the Euro area.

4 Statistical Evidence

To validate the presented theory arguing that looser margin requirements imply higher demand for houses and then a surge in housing prices and returns, a multivariate regression analysis is performed to assess the relevance of leverage ratio on the housing market. This analysis also contemplates interest rates to determine if the conventional idea that diminishing interest rates boosts housing prices is verified.

4.1 Methodology

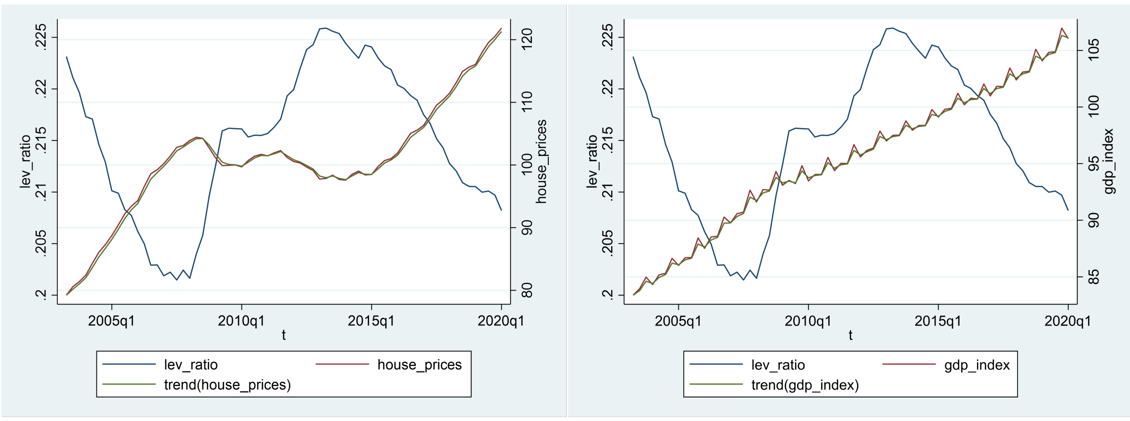
4.1.1 Data Sample

The study is solely based on the Euro area, the data for the study has been adopted from the ECB Statistical Data Warehouse. The data is secondary in nature and distributed on a cross-section of variables and time series. The data spans from the second quarter of 2003 (2003-Q2) to the 1st quarter of 2020 (2020-Q1). The sample size for each variable is 68 (n=68). The dataset under analysis is both financial and non-financial for the variables including: households' housing wealth, current prices - domestic currency (millions); loans granted to households, current prices - domestic currency (millions); residential property prices index (2015=100); GDP at market prices deflator index (2015=100); and bank interest rates charged for loans to households for house purchase with an original maturity of over five years (%). The original data extracted from the ECB Statistical Data Data Warehouse is presented in Table V, Appendix B.

4.1.2 Variables

Following the theory established in the literature and the research question under study, the following variables have been used:

- **Loans Granted to Households (**$debt$**):** The variable loans granted to households represent the loans granted in the Euro area to households and non-profit institutions serving households.
- **Households' Housing Wealth (**$collateral$**):** The variable housing wealth represents the estimated wealth of households and non-profit institutions serving households in the Euro area and accounts for the collateral held by households.
- **Residential Property Prices (**$prices$**):** The variable residential prices serve as a proxy for accounting risky asset prices used in the model presented in section



(a) (b)

Figure 5: Leverage ratio, residential property prices and GDP index

3. For the purpose, the residential property price index has been adopted for all dwelling types, new and existing.

- **GDP Deflator Index (*gdp*):** This GDP variable accounts for the economic activity including domestic (home or reference area) and is measured as a deflator index. The sample used refers to non-transformed data, neither seasonally adjusted nor calendar adjusted.
- **Interest Rate (*interest*):** The variable interest rate accounts for bank interest rates on loans to households for house purchases with an original maturity of over five years (outstanding amounts) in the Euro area.
- **Leverage ratio (*leverage*):** The variable leverage ratio used is similar to the LTV ratio. The households' leverage ratio for the Euro area has been computed by dividing housing wealth from loans granted to households as shown in equation 22.

$$leverage = \frac{debt}{collateral} \tag{22}$$

Figure 5 provides some insights into the evolution of households' leverage ratio, house prices and GDP during the last two decades. One can argue that while the time series of house prices and GDP show a clear growth trend, the same does not apply to the leverage ratio.

4.1.3 Rates of Return

In order to find the best regression possible and have a variable that fits equation 21 presented at the end of section 3, the rate of return of housing has been computed as shown in equation 23.

$$return_t = \frac{prices_t - prices_{t-1}}{prices_t} \tag{23}$$

Yet, in an attempt to remove the effects that may explain the trend shown in Figure 5a, the housing detrended return rate has been computed by subtracting the GDP growth rate from equation 23. Thus, for testing the proposed model, the dependent variable of housing detrended return rate has been computed from equation 24, as follows:

$$dreturn_t = return_t - \frac{gdp_t - gdp_{t-1}}{gdp_t} \tag{24}$$

4.1.4 Analytical Procedures

Following equation 24, the present study adopts a multivariate regression approach to explain the proposed effects as established in section 3. To do this, the main concern is about the direction and the significance of the relationship between the leverage ratio in the period t, $t-1$ and $t-2$, and the rate of return on housing. The regression used to assess this relationship is presented in equation 25.

$$\begin{aligned} dreturn_{t,i} = {} & \beta_0 + \beta_1 \left(leverage_{t,i}\right) + \beta_2 \left(leverage_{t-1,i}\right) + \\ & + \beta_3 \left(leverage_{t-2,i}\right) + \beta_4 \left(interest_{t,i}\right) + \varepsilon_{t,i} \end{aligned} \tag{25}$$

Further, to assess the model fitness, the R-square measure of goodness of fit and analyses of variance for overall model fit were accounted for.

4.2 Regression Model Results

4.2.1 Model Assessment

Table IV shows the results for the proposed relationships and model fitness. The results suggest the model presented in equation 25 is significant at the levels of significance of 1% and 5%, except for the independent variable referring to interest rates (variable introduced as a bonus, as it does not appear in the model presented in the previous section). Further, the R-square for the model is 0.55, suggesting that the independent variables, including leverage ratio for the current quarter and the previous two and interest rates,

Table IV: Regression Model Results

VARIABLES	Housing Detrended Return Rate
Leverage Ratio	-8.570***
	(1.097)
Leverage Ratio (-1)	10.637***
	(1.937)
Leverage Ratio (-2)	-2.477**
	(1.084)
Interest Rates	-0.000363
	(0.00123)
Constant	0.0815**
	(0.0404)
Observations	66
R-squared	0.550
Adjusted R-squared	0.519

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

explain a 55% variation in the housing return rate. Both the level of significance and the R square measure presented suggest that equation 25 provides a reasonable fit for explaining the proposed relationships.

4.2.2 Relationships Estimation

The results shown in Table IV suggest there is a significant negative effect of the leverage ratio on the housing return rate ($\beta_1 = -8.570, \text{ p-value } < 0.01$). This result does not support the proposed effect. However, one can derive new conclusions from the lagged leverage ratio independent variables, since the results show an even stronger relationship between leverage ratio in the previous quarter and the housing return. As Table IV shows, there is evidence of a robust and significant positive effect of the leverage ratio at $t-1$ ($\beta_2 = 10.637, \text{ p-value } < 0.01$). Nevertheless, the same results also show a significant negative effect of the leverage ratio at $t-2$, although the impact of this latter independent variable, $leverage_{t-2}$, is weaker than the previous ones ($\beta_3 = -2.477, \text{ p-value} < 0.05$).

Further, Table IV indicates a slight and insignificant negative effect of interest rates on housing returns ($\beta_4 = -0.000363,$ p-value > 0.05).

4.2.3 Some Intuition

These results may suggest there is a delayed effect of the level of households' indebtedness in the housing return rate. This can mean that the rate of return of the housing market has a higher positive reaction from margin requirements observed at $t-1$ than at the current period, which means that the market needs some time to react to the increasing leverage level. Even so, the fact that the independent variable lagged in two periods also has significant negative effects on the housing market may call into question this theory.

Regarding interest rates, these results are in line with the literature that argues that collateral requirements may have a bigger impact on prices than current interest rates, although the effect of interest rates may as well be somewhat delayed.[8]

[8]The relationship between interest rates and the rates of return of houses has not been deepened since this is not the central point of the dissertation.

5 Conclusion

This dissertation intended to simplify the leverage cycle theory presented by Geanakoplos (2010) and confirm what is common sense for an economist. Credit is a fundamental piece on the definition of prices because of its role in the demand for assets. To do so, a two-period model is firstly presented to replicate a binomial economy where there are two different assets, a risk-less and a risky asset, where the last can have different payoffs depending on the state of nature $s = U, D$ that occurs. In the baseline model, there is no access to debt in such a way that agents need to sell the assets they do not want in order to buy what they want according to their expectations on the return of those assets. Then, credit in the same model was introduced to prove that debt results in the rise of the risky price asset. By introducing debt, the risky asset returns turns out to depend positively from agents' leverage ratio, which is measured by LTV.

To study this relationship between risky asset returns and agents' collateral rates, a multivariate regression analysis has been performed to assess the relevance of leverage ratio on the housing market. By using quarterly housing data from the ECB Data Warehouse, this paper found that, between 2003 and 2020, housing return rates in the Euro area reacted negatively to increasing households' leverage observed in the current quarter, but showed a higher positive reaction from the leverage level observed in the previous quarter. The results suggest the rate of return of the housing market may show some delay to reply to looser margin requirements. Even so, the fact that the households' leverage ratio lagging in two quarters also shows significant negative effects on the housing market may call into question this theory.

This empirical analysis did leave room for more advanced econometric analyses if one can collect microdata from commercial banks to assess what were the historical LTV's at which financial intermediaries lent to households. Future research can still target the study on the housing market to appraise the evolution of the collateral requirements before and after the bust of the subprime crisis and check the behavior of house prices and economic growth.

The relevance of the leverage cycle theory comes from the fact that it gives a very reasonable explanation for the mechanism that triggered the financial crisis of 2008-09. This theory shows how a long period of relatively stable economic performance, as the Great Moderation, together with financial innovation, prompted excessive debt and looser credit standards demanded by credit institutions. These tendencies raised house prices and increased economic activity but made the economy more vulnerable to negative shocks. When bad news arrived, an abrupt turnaround in house prices and a tightening of collateral rates demanded by lenders was observed. This tightening created a feedback effect which further decreased housing prices and spilled over these deflationary pressures to other markets harming economic activity.

Regarding the model presented in this paper, an interesting further investigation could be to find the dynamics introduced with a model with more than two periods and find if the returns of investors increase over time when the price of the risky asset, used as collateral, decreases such that optimistic agents try to seize the opportunity to buy more of those assets, expecting its price will grow in the future. It should be possible to link this type of model to a social welfare model where the inequalities would increase as the leverage requirements varies because of the stronger ability of investors to hold risky and more valuable assets (mainly in the long-run). In a model with more periods of time, an increase in the uncertainty of the future value of the risky asset can also be observed, since in each period of time, there is more than one state of nature, which should result in more volatility in the asset market and thereby higher margin requirements by lenders.

Another possibility for future investigation on the presented model is to consider more than two states of nature where defaults are not ruling out and agents can choose to trade a contract where the bad outcome is not totally secured by the collateral. In this model, equilibrium should adjust to a lower price of loans to compensate lenders for the higher expected loss from default.

Finally, one could introduce short-selling in the model by simplifying what is presented in Geanakoplos (2010) and Fostel and Geanakoplos (2012), and see its effect on asset prices (it is expected to decrease the prices).

To conclude, this dissertation showed that there is another variable different than interest rates that has a crucial role in assessing the ability of an economic agent to borrow, which is the collateral rate required in a given debt contract. This variable is specifically relevant given the looser credit conditions provided by credit institutions prior to the subprime crisis.

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A Appendix

A.1 Solution to the baseline model

A.1.1 Solving the agent's problem

We assume that the solution for c_s^a is interior. In this case, the Lagrangean for any agent $a \in [0,1]$ is

$$\mathcal{L}^a = \pi_U^a c_U + \pi_D^a c_D + \lambda_0^a \left(e^F - f + p(e^M - m) \right) + \sum_{s \in \{U,D\}} \lambda_s^a \left(f v^F + m v_s^M - c_s \right) + \eta^{F,a} f + \eta^{M,a} m$$

The first-order optimality conditions become

$$\lambda_s^a = \pi_s^a, \; s = U, D$$
$$\lambda_0^a = (\lambda_U^a + \lambda_D^a) v^F + \eta^{F,a}$$
$$p\lambda_0^a = \lambda_U^a v_U^M + \lambda_D^a v_D^M + \eta^{M,a}$$

the budget constraints

$$0 = (f^a - e^F) + p(m^a - e^M) \tag{26}$$
$$c_s^a = f^a v^F + m^a v_s^M, \; s = U, D \tag{27}$$

and complementarity slackness equations are

$$\eta^{F,a} f^a = 0, \; \eta^{F,a} \geq 0, \; f^a \geq 0 \tag{28}$$
$$\eta^{M,a} m^a = 0, \; \eta^{M,a} \geq 0, \; m^a \geq 0 \tag{29}$$

Using the fact that $\pi_U^a + \pi_D^a = 1$, then the optimality conditions are

$$\lambda_0^a = v^F + \eta^{F,a} \tag{30}$$
$$p\lambda_0^a = \pi_U^a v_U^M + \pi_D^a v_D^M + \eta^{M,a} \tag{31}$$

From those conditions, we can split agents into two groups: agents that buy the risk-free asset and sell their initial endowment of the risky asset (group g) and agents that buy the risky asset and sell their initial endowment of the risk-free asset (group n).

A.1.2 General Public

A representative member of the "general public" sells its endowment of the risky asset and buy the risk-free asset, such that

$$f^g > 0,\ \eta^{F,g} = 0$$

and

$$m^g = 0,\ \eta^{M,g} > 0.$$

Then

$$\lambda_0^g = v^F,\ p\lambda_0^g > \pi_U^g v_U^M + \pi_D^g v_D^M = \mathbb{E}^g[v^M]$$

implying that equations (30) and (31) become

$$pv^F > \mathbb{E}^g[v^M] \tag{32}$$

From equations (26) and (27), we obtain their demand for the risk-free asset

$$f^g = e^F + pe^M$$

and consumption, which is state-independent

$$c_s^g = f^g v^F,\ s = U, D.$$

A.1.3 Investors

"Investors" sell their endowment of the risk-free asset such that it is optimal for them to give a zero position on the risk-free asset implying

$$f^i = 0,\ \eta^{F,i} > 0$$

and

$$f^i > 0,\ \eta^{M,i} = 0$$

Then equation (30) and (31) become

$$\lambda_0^i > v^F,\ p\lambda_0^i = \pi_U^i v_U^M + \pi_D^i v_D^M = \mathbb{E}^i[v^M]$$

which is equivalent to saying that for the investor we have

$$pv^F < \mathbb{E}^i[v^M] \tag{33}$$

From equations (26) and (27), we obtain their demand for the risky asset

$$m^i = \frac{e^F + pe^M}{p}$$

and consumption, which is state-dependent

$$c_s^i = m^i v_s^M, \ s = U, D.$$

A.1.4 The neutral buyer

Considering the two conditions, (32) and (33), and by continuity, this means that there is a neutral agent with index $a = n \in (0, 1)$ that is indifferent between investing in the risk-less and the risky asset, such that

$$pv^F = \pi_U^n v_U^M + \pi_D^n v_D^M = \mathbb{E}^n[v^M] \tag{34}$$

where π_s^n for $s = U, D$ are the probabilities associated with the neutral buyer.

A.1.5 Equilibrium price of the risky asset

Knowing the demand of agent a for the assets is given by (7) and (8), if we run the asset market-clearing conditions (2) and (3), the equilibrium conditions for the asset markets become

$$n\left(e^F + pe^M\right) = e^F$$

and

$$(1-n)\left(\frac{e^F + pe^M}{p}\right) = e^M.$$

By Walras's Law, one of these conditions is redundant. Thus we get the equilibrium price of the risky asset M given by

$$p = \left(\frac{1-n}{n}\right)\epsilon$$

where ϵ is the relative supply of the risk-free asset relative to the risky asset

$$\epsilon \equiv \frac{e^F}{e^M}.$$

A.1.6 Equilibrium participation rate

In order to be able to determine n, it is assumed that $\pi_s^a = \pi_s(a)$, for $s = U, D$, meaning the idiosyncratic probabilities of agents are function of a.

Using this assumption and substituting the equilibrium asset price (9) in equation (34), then n is determined endogenously from

$$n^* = \{n \in (0,1): \; (1-n)\epsilon = n\left(\pi_U^n(n)\delta_U + \pi_D^n(n)\delta_D\right)\} \tag{35}$$

with δ being the risk premium in the multiplicative form

$$\delta \equiv (\delta_U, \delta_D) = \left(\frac{v_U^M}{v^F}, \frac{v_D^M}{v^F}\right)$$

where $\delta_U > \delta_D$. Indeed, n is a fixed point of equation (35).

Assuming $\pi_U^n = n$ and $\pi_D^n = 1 - n$ as the form of the probabilities, we can solve equation (35) for $n \in (0, 1)$ and get the equilibrium population split as presented in equation (13):

$$n^* = n(\epsilon, \delta_U, \delta_D) = -\frac{\epsilon + \delta_D - \sqrt{(\delta_D - \epsilon)^2 + 4\,\epsilon\,\delta_U}}{2\,(\delta_U - \delta_D)}$$

A.2 Solution to the model with borrowing

A.2.1 Solving the agent's problem

We assume that the solution for c_s^a is interior. In this case the Lagrangean for any agent $a \in [0, 1]$ is

$$\begin{aligned}\mathcal{L}^a = \pi_U^a c_U^a + \pi_D^a c_D^a + \lambda_0^a\left(e^F - f + p(e^M - m) + \theta\psi\right) + \\ + \sum_{s\in\{U,D\}} \lambda_s^a\left(f^a v^M + m^a v_s^M - \min\{d_s, v_s^M\}\psi^a - c_s^a\right) + \\ + \eta^{F,a}x + \eta^{M,a}m + \mu^a\left(m^a - \max\{0, \psi^a\}\right)\end{aligned}$$

The first-order optimality conditions become

$$
\begin{aligned}
\lambda_s^a &= \pi_s^a,\ s = U, D \\
\lambda_0^a &= (\lambda_U^a + \lambda_D^a)v^F + \eta^{F,a}, \\
p\lambda_0^a &= \lambda_U^a v_U^M + \lambda_D^a v_D^M + \eta^{M,a} + \mu^a, \\
\theta\lambda_0^a &= \lambda_U^a \min\{d_U, v_U^M\} + \lambda_D^a \min\{d_D, v_D^M\} + \mu^a \frac{\partial \max\{0, \psi^a\}}{\partial \psi^a}
\end{aligned}
$$

the budget constraints

$$
\begin{aligned}
\theta\psi^a &= (f^a - e^F) + p(m^a - e^M) && (36) \\
c_s^a &= f^a v^F + m^a v_s^M - \min\{d_s, v_s^M\}\psi^a,\ s = U, D && (37)
\end{aligned}
$$

and complementarity slackness equations are

$$
\begin{aligned}
\eta^{F,a} f^a &= 0,\ \eta^{F,a} \geq 0,\ f^a \geq 0 && (38) \\
\eta^{M,a} m^a &= 0,\ \eta^{M,a} \geq 0,\ m^a \geq 0 && (39) \\
\mu^a (m^a - \max\{0, \psi^a\}) &= 0,\ \mu^a \geq 0,\ m^a \geq \max\{0, \psi^a\} && (40)
\end{aligned}
$$

Using the fact that $\pi_U^a + \pi_D^a = 1$, then the optimality conditions are

$$
\begin{aligned}
\lambda_0^a &= v^F + \eta^{F,a} && (41) \\
p\lambda_0^a &= \mathbb{E}^a[v^M] + \eta^{M,a} + \mu^a && (42) \\
\theta\lambda_0^a &= \mathbb{E}^a[\min\{d, v^M\}] + \mu^a \frac{\partial \max\{0, \psi^a\}}{\partial \psi^a} && (43)
\end{aligned}
$$

where

$$\mathbb{E}^a[v^M] = \pi_U^a v_U^M + \pi_D^a v_D^M$$

and

$$\mathbb{E}^a[\min\{d, v^M\}] = \pi_U^a \min\{d_U, v_U^M\} + \pi_D^a \min\{d_D, v_D^M\}$$

Next, we use the same approach as in the model without borrowing.

A.2.2 General Public

In this case, we have

$$f^g > 0,\ \eta^{F,g} = 0$$

and

$$m^g = 0,\ \eta^{M,g} > 0.$$

Next, we deal with the case in which the representative members of the public buy promises (i.e., when they are lenders) that is $\psi^g < 0$ and therefore $\frac{\partial \max\{0,\psi^a\}}{\partial \psi^a} = 0$.[9] Then, from (43), we know that

$$\lambda_0^g = v^F,\ p\lambda_0^g > \pi_U^g v_U^M + \pi_D^g v_D^M = \mathbb{E}^g[v^M]$$

implying that equations (41) and (42) writes as

$$pv^F > \mathbb{E}^g[v^M]$$

which is equal to equation (32) in the previous model, and equation (43) writes as

$$\theta v^F = \mathbb{E}^g[\min\{d, v^M\}] \tag{44}$$

From equations (36) and (37), we obtain the demand for the risk-free asset

$$f^g = e^F + pe^M + \theta\psi^g$$

and the consumption plan for the public becomes

$$c_s^g = f^g v^F - \min\{d_s, v_s^M\}\psi^g,\ s = U, D.$$

A.2.3 Investors

For a representative investor, we have

$$f^i = 0,\ \eta^{F,i} > 0$$

and

$$m^i > 0,\ \eta^{M,i} = 0.$$

Then, similarly to the previous model

$$\lambda_0^i = v^F + \eta^{F,i} > v^F$$

As investors sell promises since they are borrowers, we consider the case $\psi^i > 0$.

[9] The case $\psi = 0$ corresponds to the model without access to credit.

However we have two possibilities:

1. The case $\psi^i < m^i$ and therefore $\mu^i = 0$. In this case, the other conditions are

$$\begin{aligned} p\lambda_0^i &= \mathbb{E}^i[v^M] \\ \theta\lambda_0^i &= \mathbb{E}^i[\min\{d, v^M\}] \\ m^i &= \frac{e^F + pe^M + \psi^i\theta}{p} \\ c_s^i &= m^i v_s^M - \min\{d_s, v_s^M\}\psi^i,\ s = U, D \end{aligned}$$

2. The case $\psi^i = m^i$ and therefore $\mu^i > 0$. In this case the other conditions are

$$\begin{aligned} p\lambda_0^i &= \mathbb{E}^i[v^M] + \mu^i \\ \theta\lambda_0^i &= \mathbb{E}^i[\min\{d, v^M\}] + \mu^i \\ m^i &= \frac{e^F + pe^M}{p - \theta} \\ c_s^i &= m^i \left(v_s^M - \min\{d_s, v_s^M\}\right),\ s = U, D \end{aligned}$$

Then the following incentive conditions are obtained:

1. In the case $\psi^i < m^i$,

$$pv^F < \mathbb{E}^i[v^M] \tag{45}$$

$$\theta v^F < \mathbb{E}^i[\min\{d, v^M\}] \tag{46}$$

which holds if and only if $\theta\mathbb{E}^i[v^M] = p\mathbb{E}^i[\min\{d, v^M\}]$, i.e., when the investor expects a return of the risky asset M equal to the return of the promise he sold.

2. In the case $\psi^i = m^i$, we get a unique incentive condition

$$(p - \theta)\, v^F < \mathbb{E}^i[v^M] - \mathbb{E}^i[\min\{d, v^M\}] \tag{47}$$

and

$$\mu^i = \frac{\theta\mathbb{E}^i[v^M] - p\mathbb{E}^i[\min\{d, v^M\}]}{p - \theta}$$

which implies that this case only holds if $\theta\mathbb{E}^i[v^M] > p\mathbb{E}^i[\min\{d, v^M\}]$,i.e., when the investor expects that the return of the risky asset M will be higher than the return of the promise he sold to the general public.[10]

[10]Note that $p > \theta$ needs to hold in this economy since that $leverage_d \in [0, 1]$.

A.2.4 Existence of general equilibrium

Comparing conditions (44) for the public and (46) or (47) for the investor, it makes sense to assume that a contract can only be drawn when the expectations for the return of the promise for the public and investors meet each other, such that

$$\mathbb{E}^g[\min\{d, v^M\}] = \mathbb{E}^i[\min\{d, v^M\}]$$

Since promises are assumed to be non-contingent, i.e, $d_U = d_D = d$, if we also assume that $v_D^M \leq d < v_U^M$, then the previous condition writes

$$\pi_U^g d + \pi_D^g v_D^M = \pi_U^i d + \pi_D^i v_D^M.$$

Substituting π_D^a by $1 - \pi_U^a$ for $a = g, i$ in the previous condition, we derive that $d = v_D^M$, meaning that there is only equilibrium in the promises market if the payoff of the promise equals the collateral value if the worst scenario occurs, i.e., if d equals v_D^M. As a consequence, equation (44) becomes

$$\theta v^F = v_D^M \tag{48}$$

which determines the equilibrium price of promise d, θ^*, meaning it equals the risk premium at the bad state δ_D. In addition, the related condition for the investor (46) becomes $\theta v^F < v_D^M$ for the case $\psi^i < m^i$ which cannot hold in equilibrium. Therefore, in equilibrium, the investor will choose the case $\psi^i = m^i$ and equation (47) becomes $(p - \theta)\, v^F < \mathbb{E}^i[v^M] - v_D^M$. Using the equilibrium condition (48), then we have

$$pv^F < \mathbb{E}^i[v^M]$$

which is equal to the equation (33) in the model without borrowing.

A.2.5 The neutral agent

Because we have the same incentive conditions (32) and (33) as in the previous model for the public and investors respectively, the incentive condition for the neutral buyer is identical to (34):

$$pv^F = \mathbb{E}^n[v^M]$$

A.2.6 Equilibrium price of the risky asset

Knowing the demand of agent a for the assets is given by (15) and (16) and that in equilibrium, investors will choose the case where the number of promises they sell will be equal to their holdings of M such that $\psi^i = m^i$, if we run the market-clearing conditions (2), (3) and (**??**), the equilibrium conditions for asset and promise markets become

$$n\left(e^F + p\,e^M + \theta^*\psi\right) = e^F$$

$$(1-n)\left(\frac{e^F + p\,e^M}{p - \theta^*}\right) = e^M$$

and

$$n\,\psi^g + (1-n)\,\psi^i = 0$$

By Walras' Law, only two equations from the equilibrium conditions are independent. Using those conditions, we can determine the market equilibrium for the price of the risky asset

$$p = \frac{1-n}{n}\epsilon + \frac{v_D^M}{n\,v^F}$$

A.2.7 Equilibrium participation rate

Assuming the idiosyncratic probabilities of agents are a function of a such that $\pi_s^a = \pi_s(a)$, for $s = U, D$ and $\pi_U^n = n$ and $\pi_D^n = 1 - b$ as the the form of of the probabilities, we can again substitute the new equilibrium asset price (18) in equation (34) in order to get the equilibrium population split as presented in equation (20):

$$n^* = n(\epsilon, \delta_U, \delta_D) = -\frac{\epsilon + \delta_D - \sqrt{(\delta_D + \epsilon)\,(4\delta_U - 3\delta_D + \epsilon)}}{2\,(\delta_U - \delta_D)}$$

B Appendix

Table V: ECB Statistical Data Warehouse Original Data

Quarter	Households' housing wealth, current prices, domestic currency (millions)	Loans granted to households, current prices, domestic currency (millions)	Residential property prices index (2015Q4=100)	Gross domestic product at market prices deflated index (2016Q1=100)	Bank interest rates charged for loans to households for house purchase with an original maturity of over five years (%)
2003-Q1	17349139,23	3892690,25	77,8	82,66	5,55
2003-Q2	17750389,72	3960766,5	79,22	83,38	5,4
2003-Q3	18204242,33	4025003	80,58	83,95	5,26
2003-Q4	18670386,61	4100864,75	81,4	84,98	5,15
2004-Q1	19082374,23	4146810,75	82,42	84,32	5,05
2004-Q2	19627603,53	4260814	84,3	85,18	4,97
2004-Q3	20223698,68	4340201	86,06	85,32	4,9
2004-Q4	20816131,94	4431912	87,29	86,64	4,84
2005-Q1	21376863,87	4491717,5	88,74	86,01	4,78
2005-Q2	21996302,33	4616801	90,44	86,69	4,67
2005-Q3	22607570,97	4708961	92,2	86,71	4,59
2005-Q4	23178044,32	4815103,5	93,42	88,44	4,51
2006-Q1	23800993,8	4907075	94,32	87,52	4,52
2006-Q2	24531071,06	5028460,5	96,58	88,52	4,55
2006-Q3	25219686,27	5117426,5	98,6	88,59	4,62
2006-Q4	25736731,72	5222960	99,25	90,28	4,7
2007-Q1	26306746,1	5310914,5	100,22	89,77	4,79
2007-Q2	26823799,62	5424711	101,43	90,58	4,86
2007-Q3	27330298,5	5506511,5	102,83	90,72	4,94
2007-Q4	27681065,57	5603545,5	103,11	92,66	5,01
2008-Q1	28010553,94	5648134	103,91	91,61	5,02
2008-Q2	28166857,69	5745129,5	104,43	92,71	5,07
2008-Q3	28156068,16	5794443,5	104,26	92,66	5,14
2008-Q4	27851537,24	5838661,5	102,72	94,31	5,09
2009-Q1	27554225,74	5858737,5	101,1	93,09	4,78
2009-Q2	27332864,13	5901862	99,91	93,52	4,46
2009-Q3	27393739,68	5922192	99,95	93,24	4,25
2009-Q4	27581633,67	5961870	100,01	94,82	4,07
2010-Q1	27639147,13	5973344,5	99,7	93,48	3,98
2010-Q2	27943655,74	6017370,5	100,72	94,04	3,84
2010-Q3	28018059,37	6038223,5	101,42	94,05	3,82
2010-Q4	28252867,06	6088166,5	101,7	95,55	3,8
2011-Q1	28228901,68	6088544,5	101,46	94,38	3,83
2011-Q2	28383230,7	6138902,5	101,94	95,03	3,86
2011-Q3	28361084,52	6156538	102,31	95,01	3,91
2011-Q4	28142741,23	6172566	101,19	96,68	3,89
2012-Q1	27986950,72	6155439	100,56	95,59	3,85
2012-Q2	27805891,73	6172830	100,32	96,17	3,75
2012-Q3	27537857,51	6163368	99,68	96,38	3,66
2012-Q4	27479303,96	6163318	99,12	97,88	3,55
2013-Q1	27155195,07	6132250	97,76	97	3,48
2013-Q2	27140862,6	6130980,5	97,86	97,5	3,42
2013-Q3	27180800,48	6131760	98,38	97,52	3,36
2013-Q4	27148149,32	6118371,5	97,7	98,78	3,31
2014-Q1	27190489,7	6102245	97,58	97,95	3,32
2014-Q2	27355759,48	6118739,5	98,52	98,37	3,28
2014-Q3	27434008,29	6117203	99,03	98,38	3,22
2014-Q4	27378266,27	6139478,5	98,42	99,78	3,13
2015-Q1	27427203,68	6145321,5	98,49	99,11	3,08
2015-Q2	27647245,15	6165510	99,77	99,8	2,98
2015-Q3	27845345,36	6188148	100,71	99,88	2,88
2015-Q4	27985887,7	6209387	101,03	101,21	2,8

Quarter	Households' housing wealth, current prices, domestic currency (millions)	Loans granted to households, current prices, domestic currency (millions)	Residential property prices index (2015Q4=100)	Gross domestic product at market prices deflated index (2016Q1=100)	Bank interest rates charged for loans to households for house purchase with an original maturity of over five years (%)
2016-Q1	28190696,58	6212696	101,91	100,2	2,75
2016-Q2	28442220,54	6258525	103,49	100,78	2,65
2016-Q3	28682392,14	6291890	105,05	100,71	2,58
2016-Q4	28855874,02	6316666	105,56	102,04	2,5
2017-Q1	29155259,72	6341649,5	106,32	100,98	2,44
2017-Q2	29522040,53	6396318,5	107,86	101,83	2,37
2017-Q3	29908686,74	6435511,5	109,55	101,8	2,32
2017-Q4	30236358,25	6479458	110,39	103,45	2,26
2018-Q1	30571311,89	6504301	111,46	102,39	2,23
2018-Q2	30969513,32	6567697	113,09	103,09	2,18
2018-Q3	31373056,49	6617123,5	114,93	103,16	2,15
2018-Q4	31674171,06	6668829	115,63	105,1	2,11
2019-Q1	31852287,66	6705933,5	116,06	104,08	2,09
2019-Q2	32276283,75	6777722	117,98	104,83	2,06
2019-Q3	32565990,11	6842044,5	119,56	104,88	2,01
2019-Q4	32898327,88	6898813,5	120,55	106,97	1,96
2020-Q1	33217895,35	6916185	121,85	106,03	1,91
2020-Q2	33703221,92	6963207,5	124,01	107,35	1,88