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Portfolio insurance – a comparison of naive versus popular strategies

Abstract

This study makes a comparison between the most popular strategies of portfolio insurance – OBPI, CPPI and SLPI strategies – using Monte Carlo simulation, assuming the underlying asset follows a geometric Brownian motion.

The authors compare them in terms of moments of the distribution of returns, performance ratios and stochastic dominance, under different market conditions.

The main result is found the naive CPPI 1 and SLPI strategies perform better in all scenarios. The authors also find that the CPPI 1 strategy stochastically dominates, on second and/or third order, the other strategies in bear market scenarios. CPPI strategies, with a multiplier higher than 1, have the worst performances and present extremely high probabilities of reaching (or being extremely close to) the floor.

When using real stock market data the results are similar, i.e. naive strategies outperform the standard portfolio insurance strategies.

Keywords: portfolio insurance, Monte Carlo simulation, constant proportion portfolio insurance (CPPI), option based portfolio insurance (OBPI), stop-loss portfolio insurance (SLPI).
JEL Classification: D14, D18, D80, G01, G11, G13, G24.

Introduction

Portfolio insurance strategies appeared at the end of the 70's in the financial industry. Leland and Rubinstein (1976) implemented the so-called option based portfolio insurance (OBPI) which combines a listed put and an investment in the underlying asset. Later, Perold (1986) introduces the constant proportion portfolio insurance (CPPI). Stop-loss portfolio insurance (SLPI) is analyzed by Rubinstein (1985) in a portfolio insurance context. Portfolio insurance had a high development in recent decades. Its popularity results from the fact that insurance strategies allow investors to limit downside risk while preserving the upward potential.

This study compares the most popular Portfolio Insurance strategies – CPPI with a multiplier higher than one and OBPI – with the naive CPPI 1 and SLPI portfolio insurance strategies. The goal is to understand whether popular strategies outperform (or not) naive strategies.

In the literature of portfolio insurance there are several other comparison studies, however naive strategies have been overlooked. Black and Rouhani (1989) compare CPPI and OBPI and find that OBPI has better performance than CPPI under a moderate market increase, but under small or large increases and market declines, the CPPI is better than OBPI. Cesari and Cremonini (2003) compare nine different strategies and conclude that CPPI has better performance in bear and no-trend markets. Bertrand and Prigent (2005) compare OBPI to CPPI assuming that the risky asset follows a geometric Brownian motion and find that OBPI dominates CPPI in terms of mean-variance, but CPPI has less downside risk and is high positively skewed. Khuman et al. (2008) show that the CPPI 3 and 5 have poorly performances compared with CPPI 1 for volatilities of the underlying asset greater than 10%. Annaert et al. (2009) compare CPPI, OBPI, SLPI and Buy and Hold strategies using a simulation from an empirical distribution. They find that a Buy and Hold strategy has higher returns than the other strategies, but there is no evidence of stochastic dominance between all strategies. Their results also suggest that a floor value of 100% should be preferred to lower values. Zagst and Kraus (2011) compared CPPI with a multiplier higher than one and OBPI strategies using stochastic dominance criteria.

Our analysis relates to the existing literature by comparing different portfolio insurance strategies based on statistics, performance measures, and stochastic dominance, in predetermined scenarios, created through Monte Carlo simulations. It goes beyond the existing literature by comparing popular strategies (sold and managed by financial institutions) with naive strategies (that are simpler, could be set up by investors). The key objective is to know which strategies are best for each scenario.

The rest of the document is organized as follows. Section 1 introduces all portfolio insurance strategies under analysis. Section 2 explains the methodology used. Section 3 presents and discusses the main results. In Section 4 we apply all studied strategies to real stock market data. Finall Section concludes the study and discusses further research.

1. Portfolio insurance strategies

A portfolio insurance strategy can be defined as an investment that guarantees a percentage of the initial investment at maturity. The investor has the ability to limit downside risk, particularly in falling markets, while allowing some participation in upside markets (Bertrand and Prigent, 2005).

Among strategies of portfolio insurance there are the option based portfolio insurance (OBPI), the constant proportion portfolio insurance (CPPI) and the stop-loss portfolio insurance (SLPI).

The original OBPI strategy introduced by Leland and Rubinstein (1976) was a static strategy based upon financial options. The investment was allocated between a risk-free investment and a call option on the underlying portfolio or between the underlying portfolio and a put option on that underlying portfolio. To build the original OBPI strategy it would be necessary to find listed options with specific strike prices and maturities for each underlying, which is often not possible. Thus, Leland and Rubinstein (1981), based on the pricing formula of Black and Scholes (1973) and Merton (1973), developed a dynamic OBPI strategy replicating the payoff of an option. The dynamic strategy allocates capital between risk-free assets and risky asset, where the proportion invested between these two assets is defined through delta hedging according to the BlackScholes model. The CPPI strategy was introduced by Perold (1986) and Black and Jones (1987) for equity instruments and later by Perold and Sharpe (1988) for fixed-income instruments. A CPPI is also a dynamic strategy that allocates the portfolio's wealth between risk-free and risky investments, where the weights depend upon the definition of a floor and a multiplier.

Finally, the SLPI strategy used by Rubinstein (1985) can be also seen as a dynamic portfolio strategy, where the initial wealth is fully invested in risky assets. If, however, the strategy decreases too much in value, becoming worth only the present value of the guaranteed capital, it will become fully in the risk-free asset. Although seen as dynamic is a very naïve strategy that only considers a single transaction that may (or may not) occur.

All the above mentioned strategies result from combining investment in the risky asset and the risk-free asset. Table 1 summarizes the investment strategies. Before implementing a strategy the investor or manager must decide the value of the floor (K), which means he has to choose what percentage of the initial investment he wants to guarantee at maturity (T).

Table 1. Portfolio insurance strategies summary

	CPPI	OBPI	SLPI
Price	$P_t^{CPPI} = ES_t^{CPPI} + EB_t^{CPPI}$	$P_t^{OBPI} = ES_t^{OBPI} + EB_t^{OBPI}$	$P_t^{SLPI} = ES_t^{SLPI} + EB_t^{SLPI}$
Definitions	floor $K_t = K_T \times e^{-r(T-t)}$ cushion $C_t = P_t^{CPPI} - K_t$ multiplier *m*	Price of ATM calls *Call*$_0$ number of options $q = \dfrac{\left(P_0^{OBPI} - Ke^{-rT}\right)}{Call_0}$ initial deposit $EB_0^{OBPI} = (1-q)Ke^{-rT}$	floor $K_t = K_T \times e^{-r(T-t)}$
Investment in risky asset	$ES_t^{CPPI} = m \times C_t$	$ES_t^{OBPI} = qS_t N(d_1)$	if $P_t^{SLPI} > K_t$ $ES_t^{SLPI} = P_t^{SLPI}$ otherwise $ES_t^{SLPI} = 0$
Investment in risk-free asset	$EB_t^{CPPI} = P_t^{CPPI} - ES_t^{CPPI}$	$EB_t^{OBPI} = qKe^{-r(T-t)}N(-d_2) + EB_0^{OBPI}e^{rt}$	if $P_t^{SLPI} > K_t$ $EB_t^{SLPI} = 0$ otherwise $EB_t^{SLPI} = K_t$

For the OBPI the proportions invested in the assets rely on the option pricing model introduced by Black and Scholes (1973) and the relevant quantities are: the price of at-the-money calls at the initial investment (*Call*$_0$), the number of ATM call options one could buy at start (*q*), and the initial deposit (EB_0^{OBPI}). In the formulas of Table 1 *N* (*x*) is the cumulative probability distribution function for a standardized normal distribution.

For CPPI strategies the investor must decide both the floor and a constant called the multiplier (*m*). At any moment, the difference between the strategy value and the floor is called the cushion. The investment in the risky asset is the cushion times the defined multiplier. The higher the multiplier, the higher is the participation in a sustained increase in the risky assets. Nevertheless if there is a sustained decrease in the risky assets, the faster the portfolio approaches the floor. Consequently the cushion approaches zero and the exposure to the underlying risky asset approaches zero too (Bertrand and Prigent, 2005). Common multiplier values are in between 3 and 7, so typically we have $m > 1$. Nonetheless, as it will become clear, the special case where we have $m = 1$ is a quite interesting naive strategy. A CPPI 1 corresponds to putting aside the present value of the future guarantee and investing only the remaining amount in the risky asset, without any reallocation up to maturity. In our analysis we consider three CPPI strategies: CPPI 1, CPPI 3 and CPPI 5.

The SLPI is the second naive strategy we consider. It initially invests the entire amount in risky assets and the investor or the manager monitors the strategy value so that if it ever touches the floor level then the total amount would be transferred to the bank account, to comply with the future guarantee.

Most of the previous comparison studies focus on the comparison between CPPI strategies with multipliers of 3 or higher with the OBPI, ignoring the naive strategies – CPPI 1 and SLPI. Here we consider these two naive obvious strategies and show they tend to outperform both the CPPI strategies with a multiplier higher than one and the OBPI.

2. Methodology

We use standard Monte Carlo simulation to evaluate five portfolio insurance strategies: CPPI 1, CPPI 3, CPPI 5, OBPI and SLPI. In the literature, 3 and 5 are the most commonly used multipliers (see for example Bouyé, 2009). We assume the existence of a risk-free asset with a constant rate of return (r). So the value of the risk-free asset (B) evolves according to $dB = Brdt$. Also, we assume the risky asset follows a geometric Brownian motion. That is, its dynamics follow:

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where μ and σ are constant and W is a Wiener process.

For the path simulation we use a simple Euler discretization of the dynamics of $ln\ S$ which is a simple generalized Wiener process,

$$d \ln S = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma dz \cdot$$

This implies that a risky asset price at time T, given its price today, is lognormally distributed (Hull, 2009).

Most of the literature on portfolio insurance relies on the Balck-Scholes model, thus on the assumption that the underlying risky price follows a geometric Brownian motion as above. See for instance Bertrand and Prigent (2005) and references therein. A more realistic alternative to the classical assumption is to use empirical distributions to simulate return, see for instance Annaert et al. (2009). Here we decided for the classical approach instead of the empirical one. Our purpose is to compare the standard portfolio insurance strategies – CPPI 3, CPPI 5 and OBPI – with the naive strategies of SLPI and CPPI 1. Assuming a geometric Brownian motion for the underlying risky asset, if anything, favors the OBPI strategy that relies on the Black-Scholes model.

We define eight scenarios which characterize eight different market conditions. These scenarios are characterized by a combination of four different expected rates of return (μ = -15%; μ = -5%; μ = 5%; μ = 15%) and two different volatilities (σ = 15%; σ = 40%) for the risky asset. The scenarios are presented in Table 2 and represent normal, bull and bear markets and high and low volatile markets. In our study we call bull-market scenarios when the expected rate of return is μ = 15%, normal market scenarios when is μ = 5%, and bear market scenarios when the expected rates of return are negative (μ = -15% or μ = -5%). We also consider two floor values – 80% and 100% – and simulate the strategies in the eight scenarios for both. For simplicity, we chose a constant risk-free interest rate of 5%. Finally, the last two parameters that must be set are the maturity of the investment period in years and the number of trading days a year. The values chosen for these parameters were, once again, the most common in literature: 5 years of the investment period (see, e.g. Cesari and Cremonini, 2003) and 252 trading days (see, e.g. Hull, 2009).

Table 2. Market scenarios

	K = 80%		K = 100%	
	σ = 15%	σ = 40%	σ = 15%	σ = 40%
μ = -15%	μ = -15%; σ = 15%	μ = -15%; σ = 40%	μ = -15%; σ = 15%	μ = -15%; σ = 40%
μ = -5%	μ = -5%; σ = 15%	μ = -5%; σ = 40%	μ = -5%; σ = 15%	μ = -5%; σ = 40%
μ = 5%	μ = 5%; σ = 15%	μ = 5%; σ = 40%	μ = 5%; σ = 15%	μ = 5%; σ = 40%
μ = 15%	μ = 15%; σ = 15%	μ = 15%; σ = 40%	μ = 15%; σ = 15%	μ = 15%; σ = 40%

For each scenario we generate 100.000 paths for the risky asset. For each path we implement the five portfolio insurance strategies: CPPI 1, CPPI 3, CPPI 5, OBPI and SLPI. In literature there is no optimal solution concerning the frequency of the rebalancing. We choose to use a daily rebalancing, which means that in all strategies the proportion invested in risky assets and risk-free assets can be changed every trading day. In real life portfolio insurance strategies either rebalance daily or weekly.

3. Results

We start by looking into the density functions of returns for each strategy in each scenario. For a full picture see Figures A.1 to A.8 in the Appendix A. Table 3 reports the first four moments of the distributions as it is standard in the portfolio insurance literature (e.g. Bertrand and Prigent, 2005). Unfortunately, whenever the focus is on value protection and upward potential, these statistics are important but not sufficient for an adequate selection (as discussed in Annaert et al., 2009). In Table 3, panel A, results show that in the four bear market scenarios all portfolio insurance strategies have higher expected returns than the underlying asset and also that expected returns are higher when we consider a floor of 100%. This conclusion is justified by the ability of portfolio insurance to limit downside risk. In general, the CPPI 1 strategy has the highest expected return (with the exception of the scenario μ = -5%; σ = 40%, in which the SLPI strategy has the high estexpected return). In bear market scenarios all strategies tend to have also higher expected returns in the scenarios with higher volatility (with the exception of the CPPI 1 strategy that remains almost unchanged). In normal market scenarios the SLPI strategy has the highest expected returns. The SLPI strategy is also the single one for which a high volatility of the underlying asset, in normal and bull market scenarios, has a strong positive effect on expected returns, while the CPPI 1 strategy remains almost unchanged and the remaining strategies have a decline in returns. In the normal and bull scenarios, only the SLPI strategy sees its expected return increase with a high value for the floor, this means that if the investor or manager choose an investment with less risk and set the floor in 100%, he obtains higher expected returns in this strategy. In the other strategies opposite tends to happen.

Table 3. Distribution of returns at maturity

Panel A. Annualized Expected Returns											
Scenarios	SLPI		OBPI		CPPI 1		CPPI 3		CPPI 5		Risky Asset
	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	
μ=-15%; σ=15%	-4.34%	0.02%	-4.34%	0.02%	-0.44%	1.99%	-3.82%	0.28%	-4.25%	0.01%	-15.00%
μ=-15%; σ=40%	-2.33%	1.24%	-2.86%	0.65%	-0.44%	1.99%	-3.10%	0.51%	-3.19%	0.47%	-15.02%
μ=-5%; σ=15%	-2.21%	0.77%	-2.31%	0.65%	1.78%	3.18%	-1.62%	1.25%	-2.23%	0.68%	-4.98%
μ=-5%; σ=40%	1.20%	3.76%	-0.08%	2.04%	1.79%	3.18%	-0.44%	1.78%	-0.49%	1.75%	-4.95%
μ=5%; σ=15%	5.18%	5.76%	4.98%	5.00%	5.00%	5.00%	5.01%	5.02%	4.99%	5.00%	5.03%
μ=5%; σ=40%	7.42%	8.74%	5.04%	5.04%	5.01%	4.99%	4.94%	4.99%	4.97%	4.94%	5.01%
μ=15%; σ=15%	14.98%	15.00%	14.67%	13.37%	9.38%	7.67%	14.83%	13.48%	14.97%	14.49%	14.99%
μ=15%; σ=40%	15.70%	16.31%	12.34%	9.92%	9.37%	7.70%	13.21%	11.28%	13.42%	11.36%	15.11%
Panel B. Annualized Volatilities (Standard Deviations)											
Scenarios	SLPI		OBPI		CPPI 1		CPPI 3		CPPI 5		Risky Asset
	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	
μ=-15%;σ=15%	1.63%	0.56%	1.61%	0.55%	2.73%	1.45%	1.80%	0.77%	1.49%	0.47%	14.96%
μ=-15%; σ=40%	10.11%	7.61%	8.38%	4.85%	7.17%	3.99%	7.98%	4.58%	8.22%	4.92%	40.06%
μ=-5%; σ=15%	7.51%	4.29%	7.24%	3.72%	3.98%	2.21%	6.23%	3.17%	6.96%	3.48%	15.00%
μ=-5%; σ=40%	16.85%	13.57%	14.16%	8.89%	10.06%	5.97%	14.27%	9.22%	14.87%	9.85%	40.00%
μ=5%; σ=15%	13.65%	11.46%	13.44%	10.23%	5.57%	3.30%	12.81%	9.06%	13.74%	11.03%	15.03%
μ=5%; σ=40%	24.51%	21.00%	21.04%	14.39%	13.54%	8.47%	22.50%	16.05%	23.43%	16.93%	40.14%
μ=15%; σ=15%	15.00%	14.72%	14.78%	13.50%	7.35%	4.71%	15.28%	14.69%	15.25%	15.76%	14.98%
μ=15%; σ=40%	31.30%	28.50%	27.61%	20.24%	17.42%	11.74%	31.21%	24.90%	32.55%	26.29%	40.10%
Panel C. Relative Skewness											
Scenarios	SLPI		OBPI		CPPI 1		CPPI 3		CPPI 5		Risky Asset
	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	
μ=-15%; σ=15%	8.77	20.11	9.04	21.77	0.83	0.95	4.85	4.88	9.19	22.44	0.01
μ=-15%; σ=40%	4.32	5.51	4.57	6.20	2.29	2.77	6.43	10.79	6.20	10.25	-0.01
μ=-5%; σ=15%	2.06	3.75	2.06	3.84	0.72	0.87	2.40	3.75	2.40	5.08	0.00
μ=-5% ;σ=40%	2.54	3.12	2.68	3.76	1.94	2.54	3.58	5.70	3.54	5.63	0.00
μ=5%; σ=15%	0.43	0.93	0.45	0.99	0.63	0.78	0.73	1.61	0.50	1.34	0.00
μ=5%; σ=40%	1.53	1.91	1.66	2.32	1.59	2.07	2.13	3.30	2.07	3.34	0.01
μ=15%; σ=15%	0.01	0.11	0.04	0.17	0.51	0.68	0.02	0.37	0.00	0.10	-0.01
μ=15%; σ=40%	0.88	1.14	1.02	1.48	1.29	1.78	1.21	1.97	1.19	1.98	0.01
Panel D. Relative Kurtosis											
Scenarios	SLPI		OBPI		CPPI 1		CPPI 3		CPPI 5		Risky Asset
	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	
μ=-15%; σ=15%	96.28	485.97	105.03	617.51	4.14	4.58	47.35	53.01	117.88	786.34	3.01
μ=-15%; σ=40%	25.34	39.84	28.38	51.06	11.42	16.10	52.36	147.76	47.22	128.27	2.98
μ=-5%; σ=15%	7.30	19.32	7.34	20.29	3.79	4.31	10.31	25.40	9.16	34.77	2.97
μ=-5%; σ=40%	9.99	13.89	11.07	20.73	8.81	14.00	17.13	40.95	16.57	38.74	3.01
μ=5%; σ=15%	2.65	3.28	2.69	3.46	3.57	3.98	2.92	5.64	2.57	4.15	2.99
μ=5%; σ=40%	4.96	6.57	5.58	9.36	6.49	9.56	7.29	14.87	6.82	14.67	2.98
μ=15%; σ=15%	2.92	2.77	2.93	2.80	3.37	3.75	2.76	2.56	2.94	2.52	2.98
μ=15%; σ=40%	3.20	3.79	3.64	5.20	5.19	7.67	3.68	6.49	3.55	6.30	3.03

The annualized volatilities are presented in Table 3, panel B. As expected, in all scenarios the strategies have higher volatility when the underlying risky asset has a volatility of 40%. Another reason for a high volatility in the strategies results in the choice of the 80% floor, with the exception of one case of CPPI 5 strategy in a single scenario (note that in scenario μ=15%; σ=15%, the volatility of the CPPI 5 strategy is just 15.25%). Therefore we can state, the more volatile the underlying asset and the smaller the floor required, the higher is the volatility of returns of portfolio insurance strategies. In general the strategies have lower volatility values when compared to the risky asset, which naturally arises from the existence of a barrier. The CPPI 1 strategy presents the lowest standard deviations for both floors in seven of the eight scenarios. On the other hand, the SLPI strategy has the highest volatility in most scenarios. The relative skewness results are presented in Table 3, panel C. A return distribution with positive skew has frequent small losses and a few extreme gains and a return distribution with negative skew has frequent small gains and a few extreme losses. The return distributions of all strategies in all scenarios generally are positively skewed and the effect of choosing a higher floor causes an increase in the skewness. According to Harvey and Siddique (2000) a larger skewness makes a protection strategy more appealing. In almost all scenarios where the underlying asset has more volatility, there is an increase in skewness, for both floors, in the three CPPI strategies, but in the remaining no longer applies this standard. There is a tendency on both floors of increased skewness in all strategies as the scenarios become more bear. The strategy CPPI 1 has the lowest coefficient of skewness in the four bear market scenarios for both floors. In general the CPPI 3 and CPPI 5 strategies have the highest values of skewness. The relative kurtosis results are presented in Table 3, panel D. A leptokurtic distribution has more returns around the mean and more returns with large deviations. The return distributions have mostly leptokurtic behavior, but there are still some cases where the distributions are considered platykurtic. In most cases the more volatile the underlying asset, the higher is the kurtosis in the three CPPI strategies, in the remaining there is no standard. In most cases the strategies obtain higher kurtosis with a floor of 100%. In general the kurtosis is higher in all strategies, for both floors, as the scenarios become more bear. The CPPI 1 strategy has the lowest coefficients in almost all bear market scenarios and in the normal and bull scenarios is the strategy that has more cases of higher kurtosis. Generally the CPPI 3 strategy has the highest kurtosis and the SLPI strategy presents in some scenarios the lowest coefficients. This leptokurtic and positive-skewed behavior lead to that most of the strategies have no extreme negative returns and if extreme returns occur it will only be positive.

3.1. Performance ratio analysis. We now evaluate portfolio insurance strategies according to the Sharpe, Sortino, Omega and upside potential ratios, frequently used in the literature. The Sharpe ratio (see Sharpe, 1994) where $\overline{r_p}$ is the expected return on the portfolio; r_f is the risk-free interest rate and σ_p is the standard deviation of returns on the portfolio. The ratio is given by:

$$Sharpe\,Ratio = \frac{\left(\overline{r_p} - r_f\right)}{\sigma_p},$$

and it can be described as the return per unit of risk. The higher the ratio, the better is the combined performance of risk and return (Bacon, 2008). Although it is a commonly used measure, in a portfolio insurance context this ratio is not necessarily an adequate performance measure, since portfolio insurers do not only care about the mean and variance of returns (Annaert et al., 2009).

The Sortino ratio (see Sortino and Price, 1994) measures the excess returns over a minimum acceptable return (*MAR*). The risk in the denominator σ_d is measured by the standard deviation of returns below the *MAR* and is defined by:

$$Sortino\ Ratio = \frac{\left(r_p - MAR\right)}{\sigma_d};$$

$$\sigma_d = \sqrt{\sum_{i=1}^{n} \frac{\min\left[\left(r_i - MAR\right), 0\right]^2}{n}}.$$

This ratio is similar to the Sharpe ratio, however the volatility is replaced by downside. Using downside risk the Sortino ratio only penalises return that falls below the *MAR*. This is an important feature as most investors consider risk as the probability of not achieving their *MAR*, which means they only fear the downside risk.

The Omega ratio (see Shadwick and Keating, 2002) is the expected gain above the *MAR* value divided by the expected loss below the *MAR* and where $[a, b]$ is the interval of returns with a cumulative distribution function $F(x)$. This ratio is defined by:

$$Omega\ Ratio = \frac{\int_{MAR}^{b}\left(1 - F\left(x\right)\right)dx}{\int_{a}^{MAR} F\left(x\right)dx},$$

and can be interpreted as the probability weighted ratio of gains to losses relative to a return threshold, in a way that splits the return into two subparts according to a minimum accepted return (*MAR*). The investor or manager should always prefer the portfolio with the highest value of Omega (Bertrand and Prigent, 2011). The main advantage of the Omega measure is that it involves all the moments of the return distribution, including skewness and kurtosis (Bacmann and Scholz, 2003).

The Upside Potential ratio (see Sortino et al., 1999) measures the average returns above the *MAR* in relation to the downside deviation, as the Sortino Ratio. This ratio is defined by:

$$Upside\ Potential\ Ratio = \frac{\frac{1}{n} \times \sum_{i=1}^{n} \max\left(r_i - MAR, 0\right)}{\sigma_d},$$

and it is an alternative to the Sortino ratio. It uses probability weighted average returns above the *MAR* and considers portfolio risk as downside deviation, penalizing the volatility below the *MAR*. An important advantage of using this ratio rather than Sortino ratio is the consistency in the use of the *MAR* for evaluating both profits and losses (Plantinga and Groot, 2001). In the literature there is no optimal value for the *MAR*, however since we are analyzing strategies with guaranteed return we have decided to use the risk-free interest rate of 5% as *MAR* (see for example Khuman and Constantinou, 2009). Another common value used by the literature is 0%, which would lead to the inability to analyze strategies with a floor of 100% since they do not have negative returns.

Sharpe and Sortino ratios are difficult to interpret when negative, which means that the expected return is lower than the risk-free interest rate (McLeod and van Vurren, 2004). This leads to these ratios are less negative in the portfolio with higher volatility, standing incorrectly as being the best portfolio performance. For this reason, in scenarios where these ratios are negative we only analyze those results with Omega and Upside Potential ratios. From Table 4, panel A, the best strategy for normal and bull market scenarios in terms of Sharpe ratio is the SLPI. On the other hand, the worst strategy is the CPPI 1 strategy. Another important result is that the SLPI strategy is the single one to have better results with a floor of 100%, while for all the other 80% performs better. In terms of Sortino ratios (Table 4, panel B) and for the same normal and bull market scenarios, the SLPI strategy has the best performance in three out of four scenarios, whereas in the remainder it is the CPPI 1 strategy that has the best performance. CPPI 5 has the worst performance. In all scenarios analyzed, all the strategies have better ratios with a floor of 100%.

Table 4. Performance ratio analysis

Panel A. Sharpe Ratios											
Scenarios	SLPI		OBPI		CPPI 1		CPPI 3		CPPI 5		Risky Asset
	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	
μ=-15%; σ=15%	-5.73	-8.89	-5.80	-9.05	-1.99	-2.08	-4.90	-6.13	-6.21	-10.62	-1.34
μ=-15%; σ=40%	-0.73	-0.49	-0.94	-0.90	-0.76	-0.75	-1.02	-0.98	-1.00	-0.92	-0.50
μ=-5%; σ=15%	-0.96	-0.99	-1.01	-1.17	-0.81	-0.82	-1.06	-1.18	-1.04	-1.24	-0.67
μ=-5%; σ=40%	-0.23	-0.09	-0.36	-0.33	-0.32	-0.30	-0.38	-0.35	-0.37	-0.33	-0.25
μ=5%; σ=15%	0.01	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
μ=5%; σ=40%	0.10	0.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
μ=15%; σ=15%	0.67	0.68	0.65	0.62	0.60	0.57	0.64	0.58	0.65	0.60	0.67
μ=15%; σ=40%	0.34	0.40	0.27	0.24	0.25	0.23	0.26	0.25	0.26	0.24	0.25
Panel B. Sortino Ratios											
Scenarios	SLPI		OBPI		CPPI 1		CPPI 3		CPPI 5		Risky Asset
	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	
μ=-15%; σ=15%	-1.00	-1.00	-1.00	-1.00	-0.97	-0.97	-0.99	-1.00	-1.00	-1.00	-0.90
μ=-15%; σ=40%	-0.82	-0.78	-0.87	-0.90	-0.84	-0.86	-0.90	-0.93	-0.89	-0.92	-0.60
μ=-5%; σ=15%	-0.88	-0.91	-0.89	-0.93	-0.86	-0.87	-0.90	-0.93	-0.89	-0.94	-0.77
μ=-5%; σ=40%	-0.46	-0.28	-0.62	-0.66	-0.60	-0.63	-0.63	-0.70	-0.62	-0.68	-0.40
μ=5%; σ=15%	0.03	0.24	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
μ=5%; σ=40%	0.35	0.97	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
μ=15%; σ=15%	8.44	10.54	7.79	8.22	10.52	11.10	8.00	10.58	7.45	8.04	8.08
μ=15%; σ=40%	2.01	3.64	1.29	1.54	1.52	1.67	1.32	1.73	1.21	1.51	0.86

Table 4 (cont.). Performance ratio analysis

Panel C. Omega Ratios											
Scenarios	SLPI		OBPI		CPPI 1		CPPI 3		CPPI 5		Risky Asset
	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	
μ=-15%; σ=15%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
μ=-15%; σ=40%	0.05	0.10	0.03	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.02
μ=-5%; σ=15%	0.02	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01
μ=-5%; σ=40%	0.20	0.36	0.14	0.16	0.15	0.18	0.12	0.13	0.13	0.14	0.08
μ=5%; σ=15%	0.73	1.02	0.68	0.75	0.86	0.91	0.70	0.74	0.68	0.71	0.65
μ=5%; σ=40%	0.71	1.23	0.49	0.59	0.63	0.76	0.43	0.47	0.43	0.45	0.32
μ=15%; σ=15%	32.09	37.60	30.46	27.75	42.01	45.83	26.33	28.54	27.31	21.02	31.09
μ=15%; σ=40%	2.43	4.08	1.72	2.04	2.54	3.01	1.43	1.60	1.28	1.34	1.33
Panel D. Upside Potential Ratios											
Scenarios	SLPI		OBPI		CPPI 1		CPPI 3		CPPI 5		Risky Asset
	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	
μ=-15%; σ=15%	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
μ=-15%; σ=40%	0.05	0.09	0.03	0.04	0.03	0.04	0.03	0.03	0.03	0.03	0.01
μ=-5%; σ=15%	0.02	0.03	0.02	0.02	0.02	0.02	0.02	0.01	0.02	0.02	0.01
μ=-5%; σ=40%	0.18	0.33	0.12	0.15	0.13	0.15	0.12	0.12	0.12	0.14	0.06
μ=5%; σ=15%	0.47	0.70	0.44	0.54	0.54	0.58	0.48	0.56	0.45	0.55	0.39
μ=5%; σ=40%	0.55	0.98	0.39	0.49	0.46	0.56	0.38	0.43	0.37	0.42	0.22
μ=15%; σ=15%	7.59	9.56	7.30	7.71	9.88	10.87	7.24	9.43	6.77	7.30	7.30
μ=15%; σ=40%	1.52	2.64	1.13	1.42	1.47	1.76	1.06	1.29	0.87	1.14	0.71

According to the Omega ratio results (Table 4, panel C) and in the first scenario (μ = -15%; σ = 15%) it is difficult to choose a strategy since they have values very close to zero. Still, in all other scenarios, the best strategies are the SLPI strategy and the CPPI 1 strategy. The SLPI strategy with a floor of 100% has the best ratio in six scenarios and, with a floor of 80%, in three scenarios. The CPPI 1 strategy with a floor of 100% has the best ratio in three scenarios and one scenario with 80% of floor. The CPPI 5 in general has the worst ratios of all strategies, particularly in normal and bull market scenarios. This is an interesting result as it shows high multipliers do not necessarily imply good performance, even in bull scenarios. The SLPI in bear market scenarios is the best strategy according to Omega ratio. In most cases the strategies have better ratios for a floor of 100%. Finally, using the Upside Potential ratio (Table 4, panel D) in the first scenario (μ = -15%; σ = 15%) it is also difficult to choose a strategy since they have values very close to zero. The SLPI strategy has the best ratios in almost all scenarios and in remaining scenarios where SLPI strategy is not the best, the CPPI 1 is. Generally the CPPI 5 has the worst ratios, especially in normal and bull market scenarios, the SLPI strategy has the best ratios in bear market scenarios and in more volatile scenarios (σ = 40%) for both floors. All the strategies have better ratios in all scenarios when the floor is 100%.

Table 5 presents the probability of each strategy reaching a portfolio value no higher than the floor value plus 5% at the end of each year, during an investment period of five years, for both floors. The probability is obtained each year by the number of times (over the 100.000 paths) the portfolio value is not higher than the floor (K) discounted at the risk-free interest rate, plus 5%. The purpose of adding to this calculation the 5% is to be able to check whether the strategies end up with a value close to the value of the floor, since the CPPI strategies never are fully invested in risk-free assets, so they never actually "touch" the floor barrier, but they can be extremely close to the guarantee. Obviously, all strategies in all scenarios, have higher probability of having a value below the floor barrier (K+5%), when they have a floor of 100%. Also, in most cases, the probabilities of all strategies also increase when the scenarios are more volatile. The strategy CPPI 5 is the one with a higher probability, for all eight scenarios, when the portfolio has a floor of 100%. For a floor of 80%, the CPPI 5 has the highest probability in scenarios where the volatility is σ = 40%. In the all four scenarios which have a lower volatility (σ = 15%) and for a floor of 80%, the SLPI and OBPI strategy have the highest probabilities. The CPPI 1 strategy has the lowest probabilities, in all scenarios and for both floors.

Table 6 presents the probability that each strategy has a value higher than a portfolio fully invested in the risk-free assets with a return of 5% per year, for both floors. As before, the probability is obtained each year by the number of paths (over 100.000 paths) that the strategy value is higher than a portfolio that is rewarded by 5% at the end of each year. The aim of this probability is to know if the portfolio insurance strategies can outperform an investment without risk. We can conclude that in the four bear market scenarios and in the two normal market scenarios with (μ = 5%) there is a much higher probability of obtaining a higher return on an investment in risk-free assets than on an investment in any one of portfolio insurance strategies, whatever the floor chosen. It is only possible to see the opposite in scenarios with an expected rate of return of 15%, i.e. in bull market scenarios. Generally the SLPI and CPPI 1 (the naive strategies) have the highest probabilities, while CPPI 3 and CPPI 5 have the lowest probabilities. The exception is the bull market low volatility scenario (μ = 15%; σ = 15%) where it is more likely the value of OBPI, CPPI 3 and CPPI 5 to be higher than the portfolio value invested in risk-free assets. The strategies with a floor of 80% have always higher probabilities than a floor of 100% and the volatility of 40% in bear market scenarios increases the probabilities while in normal and bull market scenarios has the opposite effect.

Table 5. Probability of a portfolio value close to the floor

μ = -15%; σ = 15%	K	1	2	3	4	5	μ = 5%; σ = 15%	K	1	2	3	4	5
SLPI	80%	8%	50%	79%	92%	97%	SLPI	80%	0%	3%	7%	10%	14%
	100%	53%	85%	95%	98%	99%		100%	10%	20%	26%	30%	33%
OBPI	80%	0%	21%	63%	88%	97%	OBPI	80%	0%	0%	2%	7%	14%
	100%	12%	64%	90%	98%	99%		100%	1%	6%	15%	25%	34%
CPPI 1	80%	0%	0%	0%	0%	0%	CPPI 1	80%	0%	0%	0%	0%	0%
	100%	0%	0%	0%	0%	2%		100%	0%	0%	0%	0%	0%
CPPI 3	80%	0%	4%	30%	62%	83%	CPPI 3	80%	0%	0%	0%	1%	6%
	100%	1%	30%	68%	88%	96%		100%	0%	1%	3%	7%	11%
CPPI 5	80%	1%	32%	69%	89%	96%	CPPI 5	80%	0%	1%	4%	7%	11%
	100%	26%	76%	93%	98%	100%		100%	2%	12%	21%	29%	36%
μ = -15%; σ = 40%	K	1	2	3	4	5	μ = 5%; σ = 40%	K	1	2	3	4	5
SLPI	80%	36%	59%	73%	81%	86%	SLPI	80%	20%	32%	39%	45%	49%
	100%	58%	74%	82%	87%	91%		100%	38%	47%	52%	56%	59%
OBPI	80%	2%	22%	48%	71%	87%	OBPI	80%	1%	7%	19%	32%	50%
	100%	12%	44%	68%	82%	92%		100%	5%	20%	34%	47%	61%
CPPI 1	80%	0%	0%	1%	4%	11%	CPPI 1	80%	0%	0%	0%	0%	1%
	100%	0%	2%	10%	22%	36%		100%	0%	0%	2%	4%	7%
CPPI 3	80%	15%	49%	71%	82%	89%	CPPI 3	80%	6%	23%	38%	48%	57%
	100%	36%	70%	85%	92%	95%		100%	20%	40%	56%	65%	74%
CPPI 5	80%	37%	69%	83%	90%	94%	CPPI 5	80%	21%	42%	55%	64%	70%
	100%	26%	86%	93%	96%	98%		100%	47%	66%	75%	81%	84%
μ = -5%; σ = 15%	K	1	2	3	4	5	μ = 15%; σ = 15%	K	1	2	3	4	5
SLPI	80%	2%	17%	36%	53%	65%	SLPI	80%	0%	0%	0%	0%	0%
	100%	28%	54%	69%	79%	86%		100%	3%	4%	4%	3%	3%
OBPI	80%	0%	4%	21%	44%	66%	OBPI	80%	0%	0%	0%	0%	0%
	100%	3%	28%	55%	74%	86%		100%	0%	1%	1%	2%	3%
CPPI 1	80%	0%	0%	0%	0%	0%	CPPI 1	80%	0%	0%	0%	0%	0%
	100%	0%	0%	0%	0%	0%		100%	0%	0%	0%	0%	0%
CPPI 3	80%	0%	0%	5%	15%	30%	CPPI 3	80%	0%	0%	0%	0%	0%
	100%	0%	7%	25%	44%	61%		100%	0%	0%	0%	0%	0%
CPPI 5	80%	0%	8%	26%	45%	61%	CPPI 5	80%	0%	0%	0%	0%	0%
	100%	10%	40%	64%	78%	87%		100%	0%	2%	2%	3%	3%
μ = -5%; σ = 40%	K	1	2	3	4	5	μ = 15%; σ = 40%	K	1	2	3	4	5
SLPI	80%	27%	45%	57%	65%	70%	SLPI	80%	13%	21%	24%	27%	28%
	100%	48%	61%	69%	74%	78%		100%	29%	34%	35%	36%	37%
OBPI	80%	1%	13%	32%	52%	71%	OBPI	80%	0%	3%	9%	17%	29%
	100%	8%	31%	50%	66%	80%		100%	3%	11%	20%	29%	39%
CPPI 1	80%	0%	0%	0%	1%	4%	CPPI 1	80%	0%	0%	0%	0%	0%
	100%	0%	1%	4%	10%	18%		100%	0%	0%	0%	1%	2%
CPPI 3	80%	10%	35%	54%	67%	76%	CPPI 3	80%	4%	14%	23%	29%	35%
	100%	0%	56%	72%	81%	87%		100%	13%	29%	39%	46%	51%
CPPI 5	80%	28%	56%	70%	79%	85%	CPPI 5	80%	14%	30%	39%	45%	51%
	100%	56%	77%	86%	90%	93%		100%	37%	53%	61%	66%	69%

Table 6. Probability of a portfolio value above the risk-free investment

μ = -15%; σ = 15%	K	1	2	3	4	5	μ = 5%; σ = 15%	K	1	2	3	4	5
SLPI	80%	8%	2%	1%	0%	0%	SLPI	80%	47%	46%	45%	44%	43%
	100%	8%	2%	1%	0%	0%		100%	47%	45%	45%	44%	43%
OBPI	80%	8%	2%	1%	0%	0%	OBPI	80%	46%	44%	43%	43%	42%
	100%	7%	2%	1%	0%	0%		100%	44%	42%	40%	39%	39%
CPPI 1	80%	8%	2%	1%	0%	0%	CPPI 1	80%	47%	46%	45%	44%	43%
	100%	8%	2%	1%	0%	0%		100%	47%	46%	45%	44%	43%
CPPI 3	80%	8%	2%	1%	0%	0%	CPPI 3	80%	46%	44%	42%	41%	35%
	100%	6%	1%	0%	0%	0%		100%	41%	38%	35%	33%	32%
CPPI 5	80%	8%	2%	1%	0%	0%	CPPI 5	80%	47%	45%	45%	44%	42%
	100%	7%	2%	0%	0%	0%		100%	44%	40%	37%	35%	33%
μ = -15%; σ = 40%	K	1	2	3	4	5	μ = 5%; σ = 40%	K	1	2	3	4	5
SLPI	80%	24%	16%	11%	8%	6%	SLPI	80%	42%	39%	36%	35%	33%
	100%	24%	16%	11%	8%	6%		100%	42%	39%	36%	35%	33%
OBPI	80%	22%	14%	9%	6%	4%	OBPI	80%	39%	35%	32%	30%	28%
	100%	21%	13%	8%	5%	4%		100%	39%	34%	30%	27%	25%
CPPI 1	80%	24%	16%	11%	8%	6%	CPPI 1	80%	42%	39%	36%	34%	33%
	100%	24%	16%	11%	8%	6%		100%	42%	39%	36%	35%	32%
CPPI 3	80%	21%	12%	7%	5%	3%	CPPI 3	80%	37%	32%	27%	24%	21%
	100%	15%	7%	4%	3%	2%		100%	29%	24%	19%	17%	15%
CPPI 5	80%	23%	13%	8%	5%	3%	CPPI 5	80%	40%	34%	29%	25%	22%
	100%	7%	8%	4%	3%	2%		100%	30%	23%	19%	15%	13%
μ = -5%; σ = 15%	K	1	2	3	4	5	μ = 15%; σ = 15%	K	1	2	3	4	5
SLPI	80%	23%	15%	10%	7%	5%	SLPI	80%	72%	80%	85%	88%	91%
	100%	23%	15%	10%	7%	5%		100%	72%	80%	85%	88%	91%
OBPI	80%	22%	14%	9%	6%	5%	OBPI	80%	71%	79%	84%	87%	90%
	100%	21%	13%	8%	5%	4%		100%	70%	77%	82%	86%	89%
CPPI 1	80%	23%	15%	10%	7%	5%	CPPI 1	80%	72%	80%	85%	88%	91%
	100%	23%	15%	10%	7%	5%		100%	72%	80%	85%	88%	91%
CPPI 3	80%	22%	14%	9%	6%	4%	CPPI 3	80%	72%	79%	83%	86%	89%
	100%	19%	11%	6%	4%	2%		100%	67%	73%	78%	82%	85%
CPPI 5	80%	23%	15%	10%	7%	5%	CPPI 5	80%	72%	80%	84%	88%	90%
	100%	20%	12%	7%	5%	3%		100%	69%	75%	79%	82%	85%
μ = -5%; σ = 40%	K	1	2	3	4	5	μ = 15%; σ = 40%	K	1	2	3	4	5
SLPI	80%	32%	26%	22%	18%	16%	SLPI	80%	52%	53%	53%	54%	55%
	100%	33%	26%	22%	18%	16%		100%	52%	53%	54%	54%	54%
OBPI	80%	30%	23%	18%	15%	13%	OBPI	80%	49%	49%	48%	48%	49%
	100%	29%	22%	17%	13%	11%		100%	48%	47%	47%	46%	46%
CPPI 1	80%	33%	26%	21%	18%	16%	CPPI 1	80%	52%	53%	53%	54%	55%
	100%	33%	26%	22%	18%	16%		100%	52%	53%	54%	54%	54%
CPPI 3	80%	28%	20%	15%	12%	9%	CPPI 3	80%	47%	45%	43%	42%	40%
	100%	19%	14%	10%	7%	5%		100%	38%	35%	33%	31%	30%
CPPI 5	80%	31%	23%	17%	13%	10%	CPPI 5	80%	50%	47%	45%	42%	40%
	100%	22%	14%	10%	7%	5%		100%	39%	34%	31%	28%	26%

The performance results can be summarized as follows:

♦ In *normal and bull market scenarios* and according to the four performance ratios the two best strategies are the naive – SLPI and the CPPI 1. The Sharpe ratio ranks it as the best strategy in the four scenarios, but the other three ratios are divided between the two strategies, according to the different floors or volatilities in the scenarios. However, the return distributions of these two strategies are quite different. The SLPI strategy has the highest expected returns in the four scenarios, but also has some of the highest standard deviations and some of the lowest skewness and kurtosis which means that has a lower probability to obtain extreme positive values than others. The CPPI 1 strategy has exactly an opposite behavior. This strategy has some of the lowest expected returns, but also has the lowest standard deviations and some of the highest skewness and kurtosis coefficients. Another major difference between these two strategies is the fact that the CPPI 1 has the lowest probabilities to have a portfolio value lower than the floor value plus 5% and even has some probabilities that are very close to zero. The SLPI strategy is the opposite because it has some of the highest probabilities of this happening. Regarding to the probability of strategies to obtain a higher return than an investment in risk-free asset, these two strategies have almost the same probabilities and also are the highest among all others. Concerning the choice of the floor, it seems to be preferable to choose 100% in case of the SLPI strategy, but in case of the CPPI 1 strategy the choice is not so obvious (see ratios values), if the investor would prefer to have a higher mean returns with higher risk he should prefer a floor of 80% otherwise should prefer a floor of 100%. Nonetheless, the CPPI 1 strategy is the strategy that has most of dominant cases in bull markets scenarios, on the second and third order.

♦ In *bear market scenarios* (considering only Omega and Upside Potential ratios) it is difficult to identify the best strategy because all strategies have low and similar ratios. Still, SLPI seems to be the best strategy. According to the other measures we also consider CPPI 1 a very good strategy. As in the normal and bull market scenarios the distributions of these two strategies are very different. The CPPI 1 strategy has almost the highest returns in the four scenarios and presents some of the lowest standard deviations. In terms of skewness, this strategy has the lowest coefficients and also some of the lowest coefficients of kurtosis. The SLPI strategy has some of the highest standard deviations, but generally with lower expected returns than the CPPI 1 strategy. The SLPI strategy also has coefficients of skewness and kurtosis higher than the CPPI 1 strategy. The CPPI 1 strategy, on the other hand, presents the lowest probabilities to have a portfolio value lower or equal than the floor value + 5%, as opposite to the SLPI strategy, which has some of the highest probabilities. Regarding a portfolio value higher than the value of a risk-free investment, the two strategies have similar probabilities and also the highest when compared to the non-naive strategies. The floor value of 100% should be preferred for the two strategies by the investors or managers, according to most measures. As in the normal and bull market scenarios the choice between these two strategies will depend on the preferences of investors or managers.

3.2. Stochastic dominance. Stochastic dominance is a basic concept of decision theory. The decision rule for first order stochastic dominance was introduced by Quirk and Saposnik (1962) and is also the strongest form of stochastic dominance. A random variable A stochastically dominates a random variable B at the first order if and only if the cumulative distribution function of A, denoted by F_A, is always below the cumulative distribution F_B of B (see for example Bertrand and Prigent, 2005 or Zagst and Kraus, 2011). For investments, it means that if an investment strategy stochastically dominates another, all investors that prefer more to less would always prefers the dominant strategy. The second order states that A dominates B if the sum of the cumulative distribution of A is always below of the sum of the cumulative distribution of B. For the investor, if a strategy dominates another on the second order, it means that every investor who is risk averse, prefer the dominant strategy. The third order of dominance says that A dominates B if the sum of the cumulative distribution of A is always below of the sum of the cumulative distribution of B. The investors who have decreasing risk aversion with respect to wealth, will always prefer a strategy that dominates stochastically another on third order.

We computed the cumulative distribution, the sum of cumulative distribution and the sum of cumulative distribution functions of returns of the five strategies in the eight scenarios for both floors. These functions are illustrated in Appendix B, see Figures B.1, B.2 and B.3 for a floor of 80% and in Figures B.4, B.5 and B.6 for a floor of 100%. Tables 7, 8 and 9 present the results for stochastic dominance of first, second and third order, respectively. In the stochastic dominance tables below the number '1' means that the strategy on the left column dominates the correspondent in the upper row. On the first order of dominance (Table 7) there are eight cases where investors or managers always prefer the SLPI strategy to OBPI strategy. This situation occurs with a floor of 80% in three scenarios and with a floor of 100% in five scenarios. On the second order of dominance (Table 8) the investors who are risk averse always prefer the CPPI 1 strategy in all bear market scenarios (the CPPI 1 strategy dominates all the other strategies in those scenarios for both floors). In bull market scenarios the CPPI 1 also dominates in some cases, particularly in relation to CPPI 3 and CPPI 5 strategies. However we could not state that investors or managers who are risk averse always prefer the CPPI 1 strategy in bull market scenarios. The third order of dominance results (Table 9) are similar to those obtained in the second order. The differences consist in obtaining a few more cases where the CPPI 1 strategy dominates the remaining strategies. In this order of dominance the investors or managers who have decreasing risk aversion with respect to wealth always prefer the CPPI 1 strategy in bear markets scenarios, as in the previous order, and also in one normal market bull scenario (μ = 5%, σ = 15%) for both floors. In literature, Bertrand and Prigent (2005) and Zagst and Kraus (2011) already compared CPPI with a multiplier higher than one and OBPI strategies using dominance stochastic criteria and none of them found stochastic dominance on first order. Although, Zagst and Kraus (2011) extend their analysis to the second and third order, and found that CPPI 3 stochastically dominates OBPI on the third order. Annaert et al. (2009) considered all the strategies but could not find any dominance in the three orders. The importance of the results here presented are in the inclusion of the naive strategies – CPPI 1 and SLPI – that turn out to be the dominant ones (whenever dominance exists).

Our key results are not consensual. Although there seems to be no best strategy for all possible scenarios, naive strategies seem to outperform the popular strategies of OBPI and CPPI with a multiplier higher than one. Stochastic dominance shows that investors, who have decreasing risk aversion with respect to wealth, always prefer CPPI 1. In addition, in the case of CPPIs, the higher the multiplier is, the worst seems to be performance. Bad performance of standard portfolio insurance strategies has been previously documented in the literature. Garcia and Gould (1987) refer that portfolio insurance cannot outperform static mix portfolios in the long run. Our results are also consistent with Khuman et al. (2008) who find that CPPI strategies (m > 1) perform poorly compared to the CPPI 1 strategy for volatilities of the underlying asset greater than 10%. Bertrand and Prigent (2003) also refer that the higher the multiplier is, the more the portfolio value increases in a bullish market and the nearest will be to the floor in a bearish market.

Table 7. Stochastic dominance – first order

K=80%						K=100%					
μ = -15%; σ = 15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ =-15%; σ = 15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		0	0	0	0	SLPI		0	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	0	0		0	0	CPPI 1	0	0		0	0
CPPI 3	0	0	0		0	CPPI 3	0	0	0		0
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	
μ =-15%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ = -15%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		1	0	0	0	SLPI		0	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	0	0		0	0	CPPI 1	0	0		0	0
CPPI 3	0	0	0		0	CPPI 3	0	0	0		0
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	
μ = -5%; σ =15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ = -5%; σ = 15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		0	0	0	0	SLPI		0	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	0	0		0	0	CPPI 1	0	0		0	0
CPPI 3	0	0	0		0	CPPI 3	0	0	0		0
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	
μ = -5%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ = -5%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		0	0	0	0	SLPI		1	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	0	0		0	0	CPPI 1	0	0		0	0
CPPI 3	0	0	0		0	CPPI 3	0	0	0		0
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	
μ = 5%; σ = 15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ = 5%; σ = 15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		0	0	0	0	SLPI		0	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	0	0		0	0	CPPI 1	0	0		0	0
CPPI 3	0	0	0		0	CPPI 3	0	0	0		0
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	
μ = 5%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ = 5%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		0	0	0	0	SLPI		1	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	0	0		0	0	CPPI 1	0	0		0	0
CPPI 3	0	0	0		0	CPPI 3	0	0	0		0
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	

Table 7 (cont.). Stochastic dominance – first order

K=80%						K=100%					
μ = 15%; σ = 15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ = 15%; σ = 15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		0	0	0	0	SLPI		0	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	0	0		0	0	CPPI 1	0	0		0	0
CPPI 3	0	0	0		0	CPPI 3	0	0	0		0
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	
μ = 15%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ = 15%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		0	0	0	0	SLPI		0	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	0	0		0	0	CPPI 1	0	0		0	0
CPPI 3	0	0	0		0	CPPI 3	0	0	0		0
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	

Table 8. Stochastic dominance – second order

K=80%						K=100%					
μ = -15%; σ = 15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ = -15%; σ = 15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		1	0	0	0	SLPI		0	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	1	1		1	1	CPPI 1	1	1		1	1
CPPI 3	1	1	0		1	CPPI 3	1	1	0		1
CPPI 5	1	1	0	0		CPPI 5	1	1	0	0	
μ = -15%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ = -15%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		1	0	0	1	SLPI		1	0	0	0
OBPI	0		0	0	1	OBPI	0		0	0	0
CPPI 1	1	1		1	1	CPPI 1	1	1		1	1
CPPI 3	0	0	0		1	CPPI 3	0	1	0		1
CPPI 5	0	0	0	0		CPPI 5	0	1	0	0	
μ = -5%; σ = 15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ = -5%; σ = 15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		0	0	0	0	SLPI		0	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	1	1		1	1	CPPI 1	1	1		1	1
CPPI 3	1	1	0		1	CPPI 3	1	1	0		1
CPPI 5	0	0	0	0		CPPI 5	1	1	0	0	
μ = -5%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ = -5%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		1	0	0	1	SLPI		1	0	0	0
OBPI	0		0	0	1	OBPI	0		0	0	0
CPPI 1	1	1		1	1	CPPI 1	1	1		1	1
CPPI 3	0	0	0		1	CPPI 3	0	0	0		1
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	
μ = 5%; σ = 15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ = 5%; σ = 15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		0	0	0	0	SLPI		1	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	1	1		1	1	CPPI 1	0	1		1	1
CPPI 3	0	1	0		1	CPPI 3	0	1	0		1
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	
μ = 5%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ = 5%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		0	0	0	1	SLPI		1	0	0	0
OBPI	0		0	0	1	OBPI	0		0	0	0
CPPI 1	1	1		1	1	CPPI 1	0	1		1	1
CPPI 3	0	0	0		1	CPPI 3	0	0	0		1
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	
μ = 15%; σ = 15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ = 15%; σ = 15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		1	0	0	0	SLPI		0	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	0	0		0	0	CPPI 1	0	0		0	0
CPPI 3	0	0	0		0	CPPI 3	0	0	0		0
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	

Table 8 (cont). Stochastic dominance – second order

μ = 15%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ = 15%; σ = 40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		1	0	0	1	SLPI		1	0	0	0
OBPI	0		0	0	1	OBPI	0		0	0	0
CPPI 1	0	0		1	1	CPPI 1	0	0		1	1
CPPI 3	0	0	0		1	CPPI 3	0	0	0		1
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	

Table 9. Stochastic dominance – third order

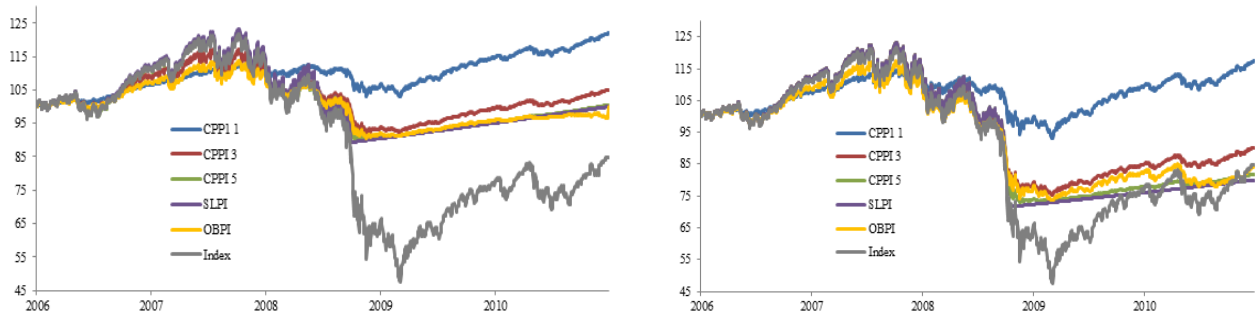
K=80%						K=100%					
μ=-15% ;σ=15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ=-15%; σ=15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		1	0	0	0	SLPI		0	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	1	1		1	1	CPPI 1	1	1		1	1
CPPI 3	1	1	0		1	CPPI 3	1	1	0		1
CPPI 5	1	1	0	0		CPPI 5	1	1	0	0	
μ=-15%; σ=40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ=-15%; σ=40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		1	0	0	1	SLPI		1	0	0	0
OBPI	0		0	0	1	OBPI	0		0	0	0
CPPI 1	1	1		1	1	CPPI 1	1	1		1	1
CPPI 3	0	0	0		1	CPPI 3	0	1	0		1
CPPI 5	0	0	0	0		CPPI 5	0	1	0	0	
μ=-5%; σ=15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ=-5%; σ=15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		0	0	0	0	SLPI		0	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	1	1		1	1	CPPI 1	1	1		1	1
CPPI 3	1	1	0		1	CPPI 3	1	1	0		1
CPPI 5	0	0	0	0		CPPI 5	1	1	0	0	
μ=-5%; σ=40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ=-5%; σ=40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		1	0	0	1	SLPI		1	0	0	0
OBPI	0		0	0	1	OBPI	0		0	0	0
CPPI 1	1	1		1	1	CPPI 1	1	1		1	1
CPPI 3	0	0	0		1	CPPI 3	0	0	0		1
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	
μ=5%; σ=15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ=5%; σ=15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		0	0	0	0	SLPI		1	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	1	1		1	1	CPPI 1	1	1		1	1
CPPI 3	0	1	0		1	CPPI 3	0	1	0		1
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	
μ=5%; σ=40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ=5%; σ=40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		0	0	0	1	SLPI		1	0	0	0
OBPI	0		0	0	1	OBPI	0		0	0	0
CPPI 1	1	1		1	1	CPPI 1	0	1		1	1
CPPI 3	0	0	0		1	CPPI 3	0	0	0		1
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	
μ=15%; σ=15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ=15%; σ=15%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		1	0	0	0	SLPI		0	0	0	0
OBPI	0		0	0	0	OBPI	0		0	0	0
CPPI 1	0	0		0	0	CPPI 1	0	0		0	0
CPPI 3	0	0	0		0	CPPI 3	0	0	0		0
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	
μ=15%; σ=40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5	μ=15%; σ=40%	SLPI	OBPI	CPPI 1	CPPI 3	CPPI 5
SLPI		1	0	0	1	SLPI		1	0	0	0
OBPI	0		0	0	1	OBPI	0		0	0	0
CPPI 1	0	1		1	1	CPPI 1	0	0		1	1
CPPI 3	0	0	0		1	CPPI 3	0	0	0		1
CPPI 5	0	0	0	0		CPPI 5	0	0	0	0	

4. Applying the strategies to World Stock Indices

After the simulation analysis previously presented, we now apply the same five portfolio insurance strategies – CPPI 1, CPPI3, CPPI 5, OBPI and SLPI – to three real stock indices for the period between 2006 and 2010 that includes the subprime crisis started in 2008. We selected three of the major world stock indices: Standards & Poor's 500 (S&P 500), the Dow Jones Euro Stoxx 50 and the Nikkei 225. The returns of these indices are used as the underlying in all strategies.

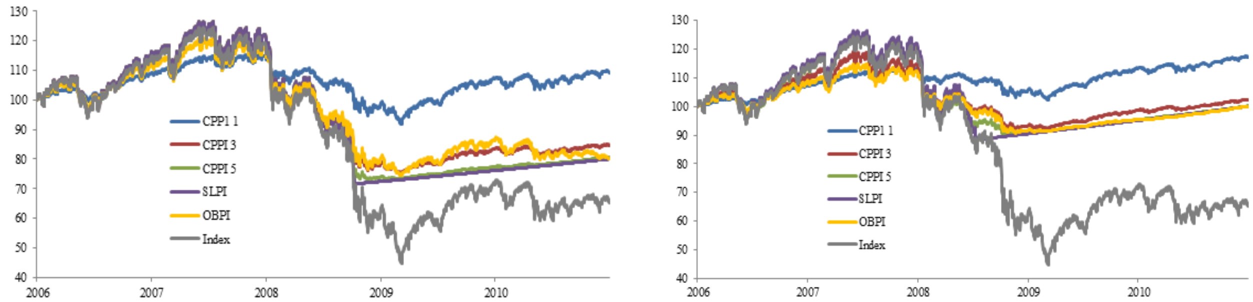
Table 10 presents the same performance measures used in Section 4.1. While in the simulations we analyzed the return distributions at maturity, where we analyze the actual historical daily logarithmic returns. The results are similar for the three indices with a negative expected return, a similar volatility and a sudden drop at the end of the last quarter of 2008, see Figures 1, 2 and 3.

As expected the SLPI strategy has the highest volatility and the CPPI 1 strategy has the lowest one. The strategies' daily returns are all left-skewed and generally have leptokurtic behavior. The omega and upside potential ratios are too low to withdraw conclusions. Nonetheless, we can say that overall the CPPI 1 strategy is the best strategy since it is the single one to have a positive expected return in the three indices for 80% floor and also has the highest expected return for a floor of 100%. We also see that the higher the multiplier of a CPPI strategy is, the sooner it approaches the floor, never to recover (or to recovering at a pace much smaller than that of the underlying). Naturally the SLPI strategy had the worst performance for all indices since, during the crisis, the floors were always reached. These results go in line with our simulation results in the scenarios that are closer to the performance of the indices and by the stochastically dominance of CPPI 1 over all strategies on the second and third order in bear markets.



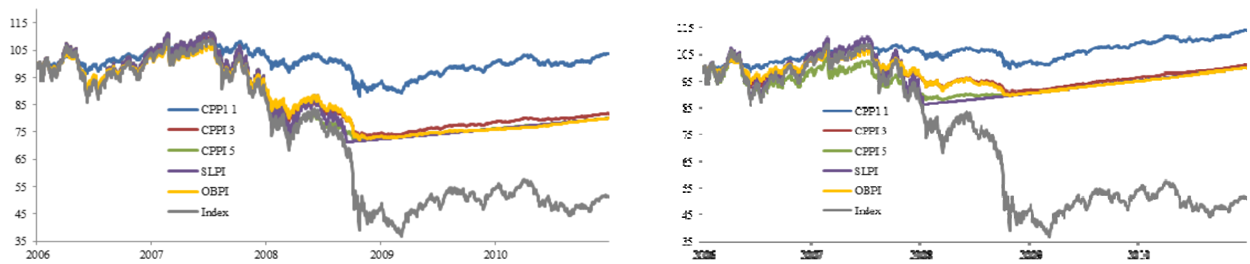
K=80% K=100%

Fig. 1. Strategies performance in S&P 500



K=80% K=100%

Fig. 2. Strategies performance in DJ EuroStoxx 50



K=80% K=100%

Fig. 3. Strategies performance in Nikkei 225

Table 10. Applying portfolio insurance to World Stock Indices

Panel A. S&P 500											
S&P 500	SLPI		OBPI		CPPI 1		CPPI 3		CPPI 5		Risky Asset
	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	
Annualized expected return	-4.47%	0.00%	-3.43%	0.00%	3.21%	3.97%	-2.08%	0.99%	-4.00%	0.09%	-3.31%
Annualized volatility	14.40%	12.94%	12.80%	7.13%	7.53%	4.25%	13.65%	7.98%	14.30%	10.61%	25.02%
Skewness	-1.84	-0.91	-0.75	-0.56	-0.33	-0.34	-0.77	-0.78	-1.31	-0.67	-0.48
Kurtosis	19.17	10.44	6.95	6.98	6.31	5.88	6.46	5.14	14.37	5.49	8.63
Sharpe ratio	-0.66	-0.39	-0.66	-0.70	-0.24	-0.24	-0.52	-0.50	-0.63	-0.46	-0.33
Sortino ratio	-1.86	-0.99	-1.66	-1.00	-0.36	-0.21	-1.39	-0.80	-1.77	-0.97	-1.58
Omega ratio	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Upside potential ratio	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.13
Panel B. DJ EuroStoxx 50											
DJ EuroStoxx 50	SLPI		OBPI		CPPI 1		CPPI 3		CPPI 5		Risky Asset
	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	
Annualized expected return	-4.38%	0.00%	-4.38%	0.00%	1.73%	3.11%	-3.28%	0.44%	-4.27%	0.02%	-8.33%
Annualized volatility	15.32%	12.79%	13.73%	7.10%	7.68%	4.33%	14.02%	8.44%	14.75%	11.37%	25.79%
Skewness	-0.74	-0.65	-0.25	-0.66	-0.05	-0.08	-0.62	-0.72	-0.59	-1.23	-0.09
Kurtosis	15.77	12.84	5.78	8.30	5.07	4.84	8.35	6.69	10.39	13.64	6.32
Sharpe ratio	-0.61	-0.39	-0.68	-0.70	-0.43	-0.44	-0.59	-0.54	-0.63	-0.44	-0.52
Sortino ratio	-1.84	-0.99	-1.84	-1.00	-0.65	-0.38	-1.63	-0.91	-1.82	-0.99	-2.52
Omega ratio	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Upside potential ratio	0.85	0.25	0.03	0.00	0.00	0.00	0.14	0.00	0.45	0.00	4.15
Panel C. performance results - Nikkei 225											
Nikkei 225	SLPI		OBPI		CPPI 1		CPPI 3		CPPI 5		Risky Asset
	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	K=80%	K=100%	
Annualized expected return	-4.59%	0.00%	-4.59%	0.00%	0.72%	2.67%	-4.13%	0.20%	-4.58%	0.00%	-13.81%
Annualized volatility	16.50%	12.38%	10.75%	6.18%	7.59%	4.21%	13.00%	7.08%	16.50%	8.95%	28.85%
Skewness	-0.56	-0.65	-0.57	-0.56	-0.36	-0.36	-0.54	-0.60	-0.56	-0.74	-0.63
Kurtosis	4.65	7.34	3.42	3.85	3.59	3.13	3.95	4.34	4.65	7.04	7.72
Sharpe ratio	-0.58	-0.40	-0.89	-0.81	-0.56	-0.55	-0.70	-0.68	-0.58	-0.56	-0.65
Sortino ratio	-1.87	-0.99	-1.89	-1.00	-0.85	-0.47	-1.80	-0.96	-1.87	-0.99	-3.51
Omega ratio	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Upside potential ratio	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.00

5. Conclusions

This study intends to contribute for a better decision on whether to invest (or not) in portfolio insurance strategies. It adds new findings and strengthens some conclusions on questions under debate by previous authors. In previous studies, the focus has been in comparing performance of popular portfolio insurance strategies such as the CPPI with multiplier higher than 1 and OBPI. Here we compare these popular strategies with naive strategies – SLPI and CPPI 1.

We find that the naive SLPI and CPPI 1 strategies outperform classic insurance strategies of OBPI, CPPI 3 or CPPI 5. The results of this study contradict some findings from the previous literature, see Annaert et al. (2009) and Cesari and Cremonini (2003). Nevertheless, they are in line with Khuman et al. (2008), who find that the CPPI 3 and 5 have worse results compared with CPPI 1 for volatilities of the underlying asset greater than 10%, and also with Annaert et al. (2009) in bear markets, when CPPI 1 provides a higher expected return but has lower ratios than the other strategies. Annaert et al. (2009) and Zagst and Kraus (2011) also reject the existence of stochastic dominance between strategies. Here we show that CPPI 1 strategy stochastically dominates all other strategies in bear markets scenarios, on second and third order. This means that investors or managers who are risk averse and have decreasing risk aversion with respect to wealth always prefer the CPPI 1 strategy in bear markets scenarios. In the definition of the floor value, our results are consistent with Annaert et al. (2009) who find that a floor value of 100% should be preferred to lower floor values. Regarding the strategies OBPI, CPPI 3 and CPPI 5, the results are not conclusive as in the literature. The performance is different according to the floor and the scenario. The strategies that seem to be the best, in general, are the CPPI 1 strategy and the SLPI strategy.

These two strategies are naive strategies that any investor can implement by himself and more complex investment strategies, whether based in options, whether in CPPI strategies with multipliers used in real life, seems to make little sense because it leads to worse performances.

Acknowledgments

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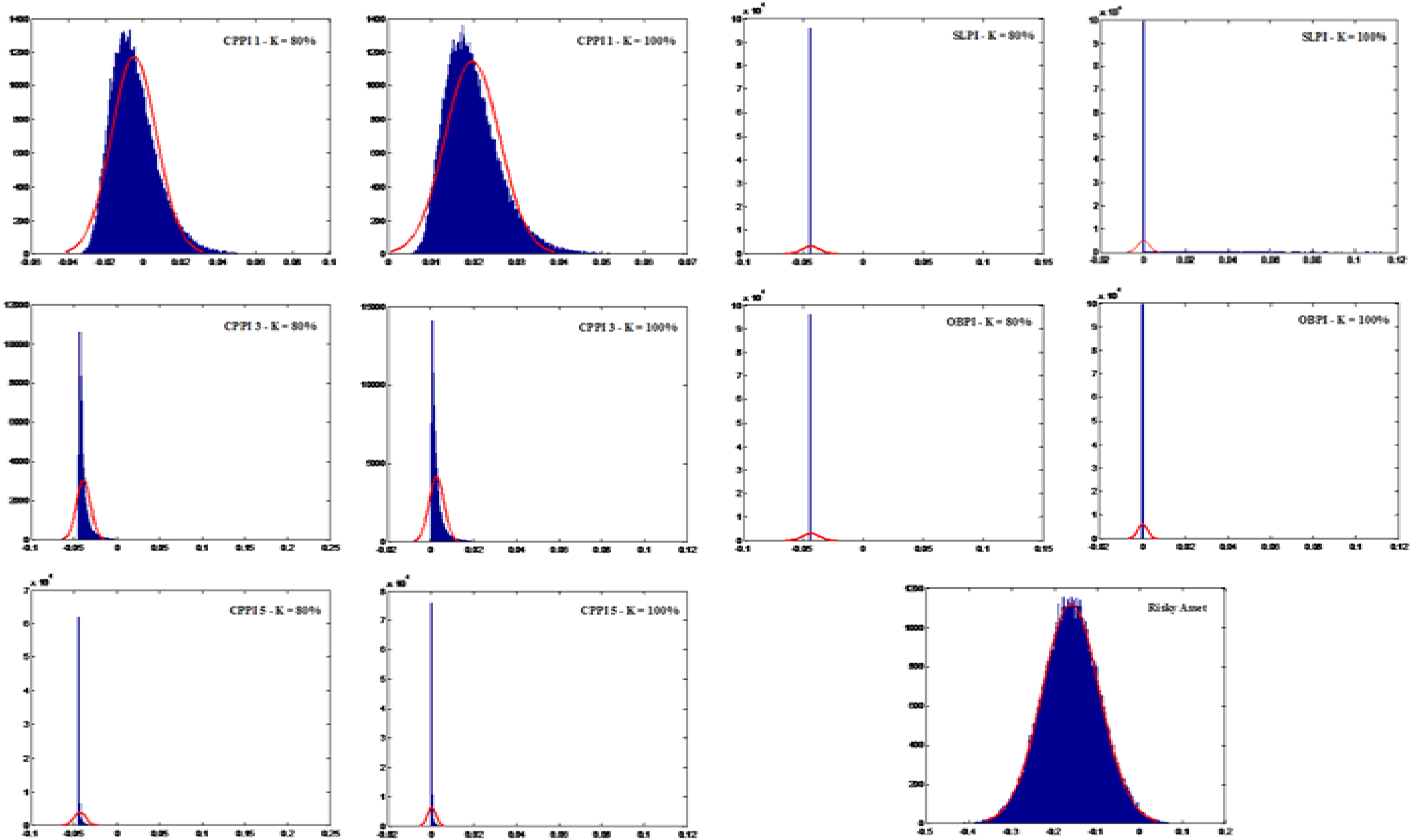
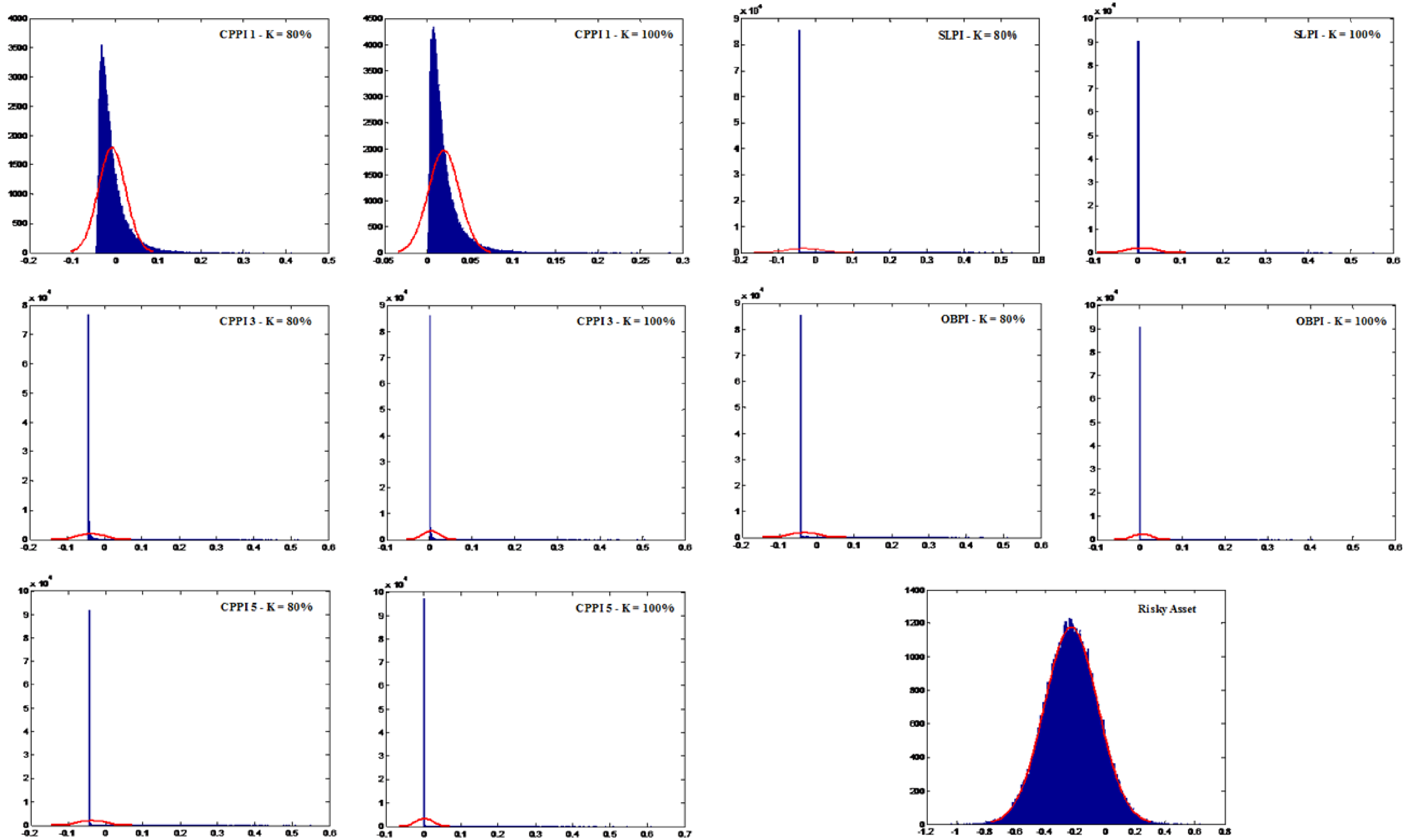
Appendix A



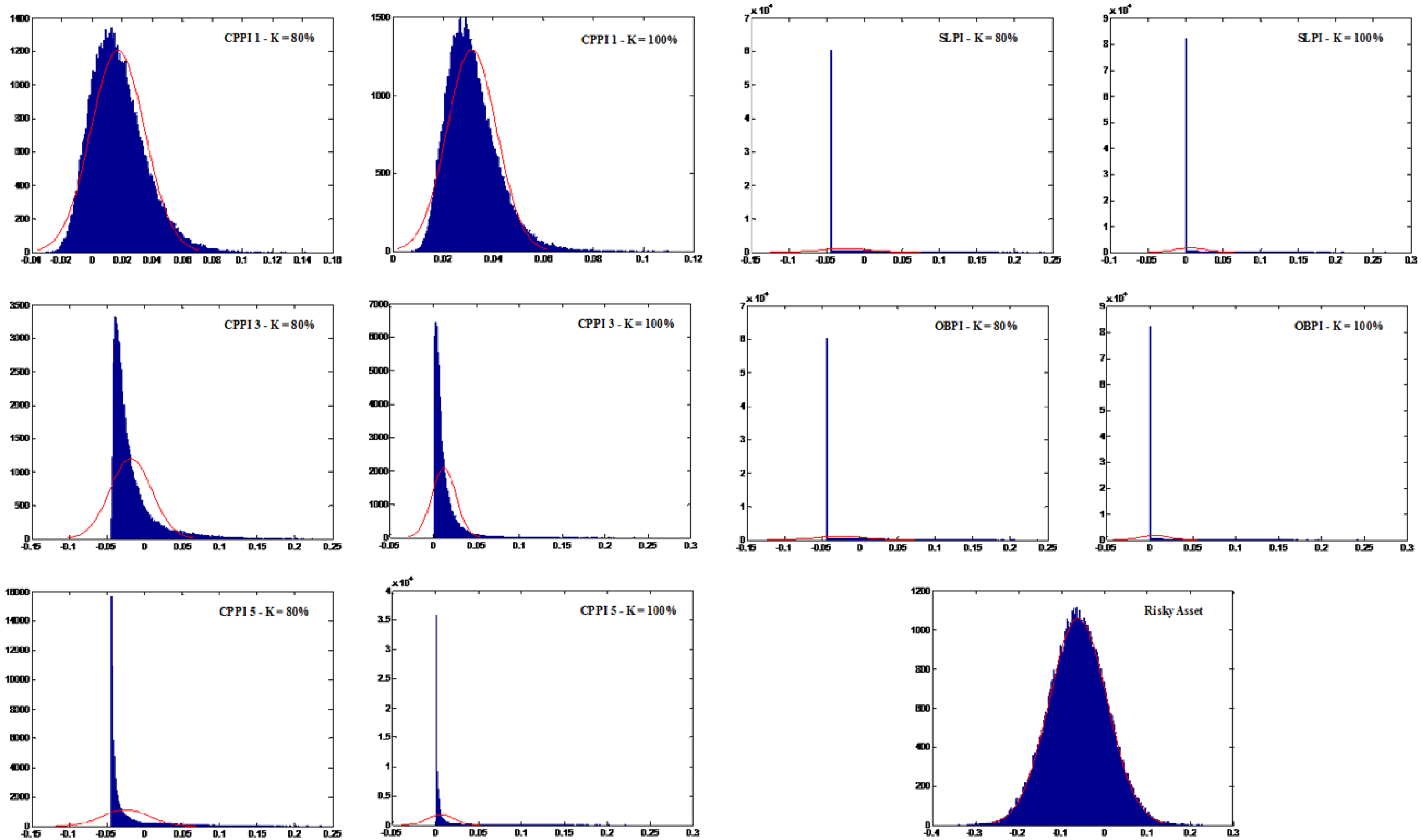
Notes: Probability density function of strategies returns – CPPI 1, CPPI 3, CPPI 5, SLPI, OBPI and Risky Asset. These results were achieved based on the following scenario: μ = -15%; σ = 15 % r = 5%; T = 5; d = 252; N = 100.000. For each strategy it was assumed a K = 80% and a K = 100%.

Fig. A.1. Return distributions with scenario: μ = -15% and σ = 15%



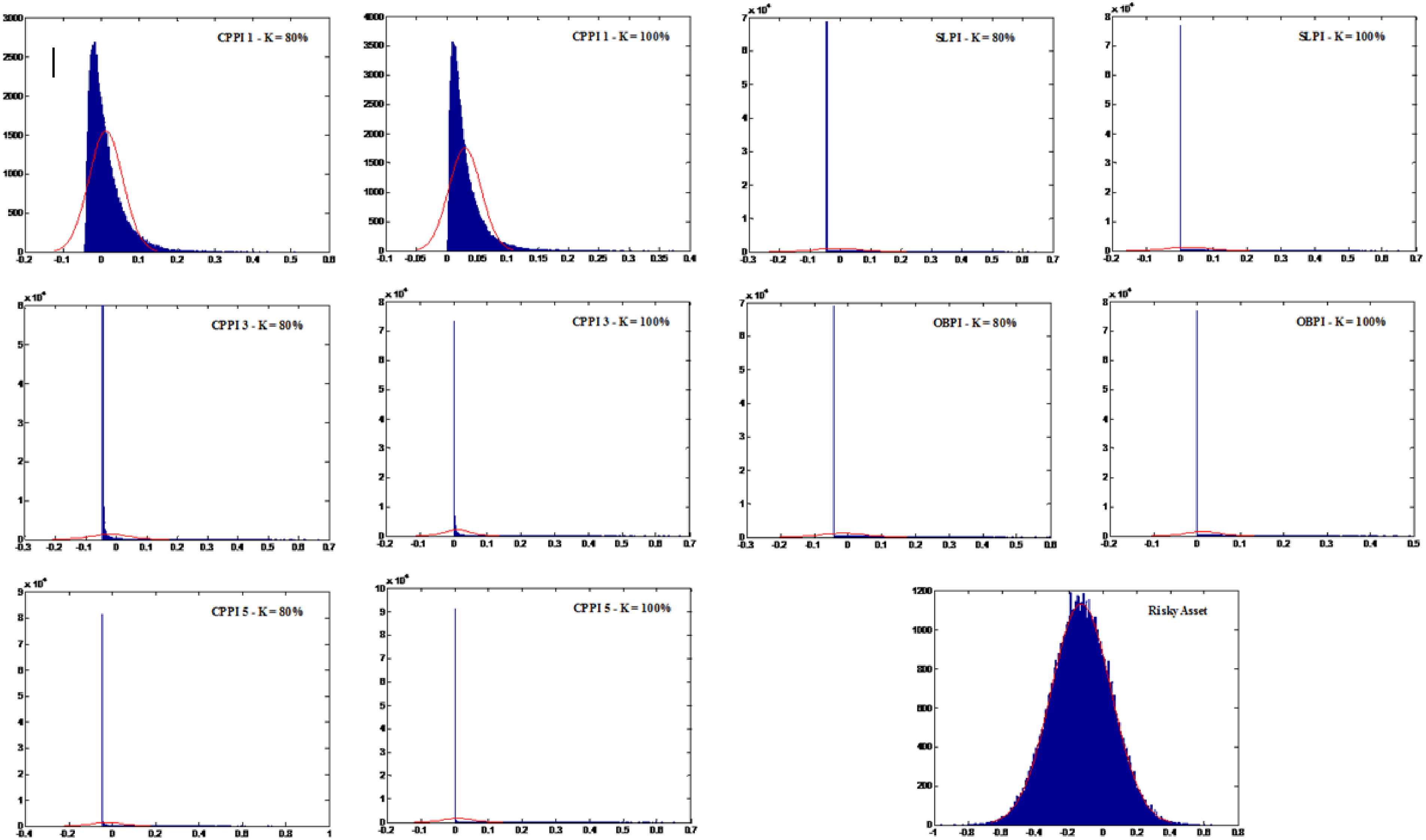
Notes: Probability density function of strategies returns – CPPI 1, CPPI 3, CPPI 5, SLPI, OBPI and Risky Asset. These results were achieved based on the following scenario: μ = -15%; σ = 40 %; r = 5%; T = 5; d = 252;   N = 100.000. For each strategy it was assumed a K = 80% and a K = 100%.

Fig. A.2. Return Distributions with scenario: μ = -15% and σ = 40%



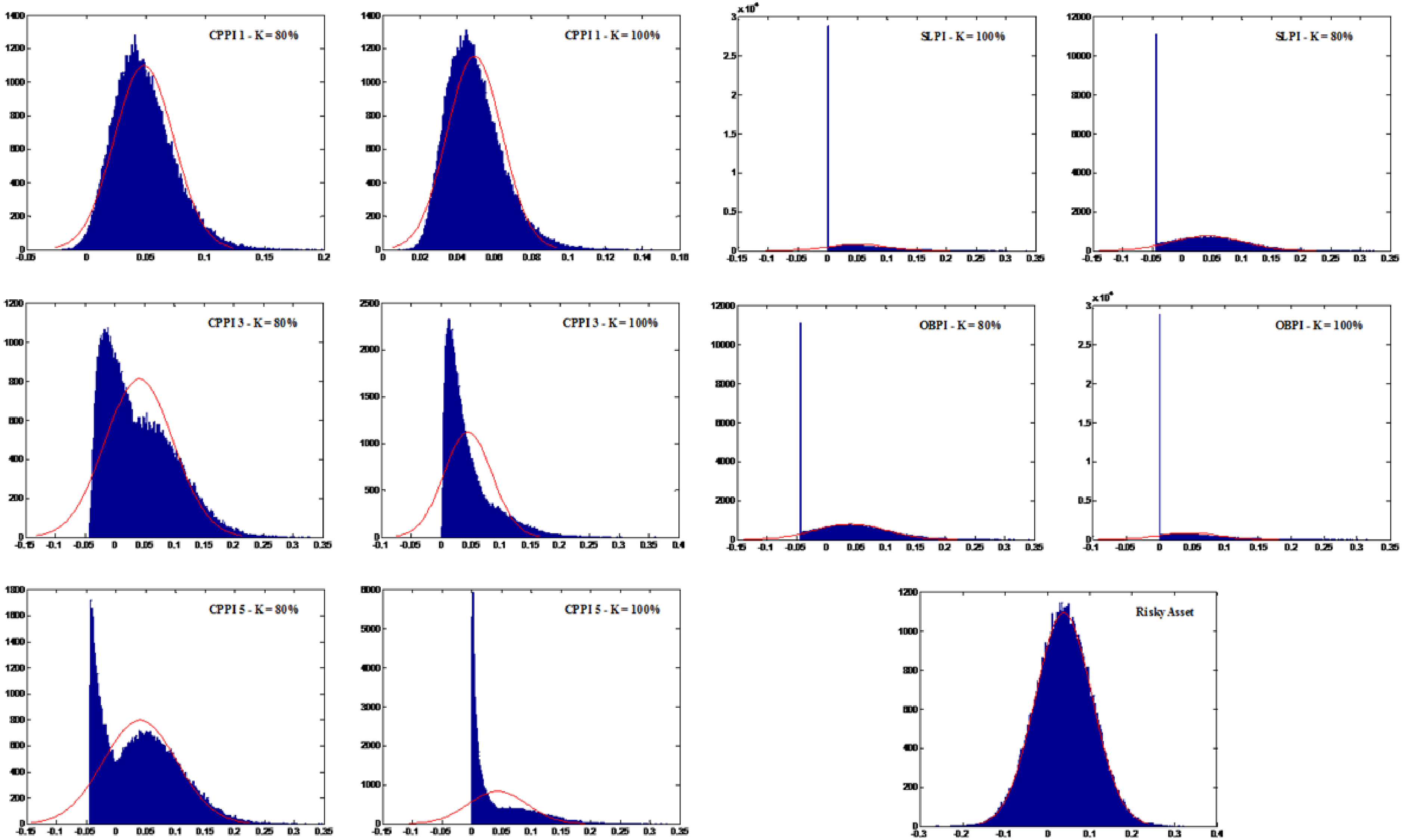
Notes: Probability density function of strategies returns – CPPI 1, CPPI 3, CPPI 5, SLPI, OBPI and Risky Asset. These results were achieved based on the following scenario: μ = -5%; σ = 15 %; r = 5%; T = 5; d = 252; N = 100.000. For each strategy it was assumed a K = 80% and a K = 100%.

Fig. A.3. Return Distributions with scenario: μ = -5% and σ = 15%



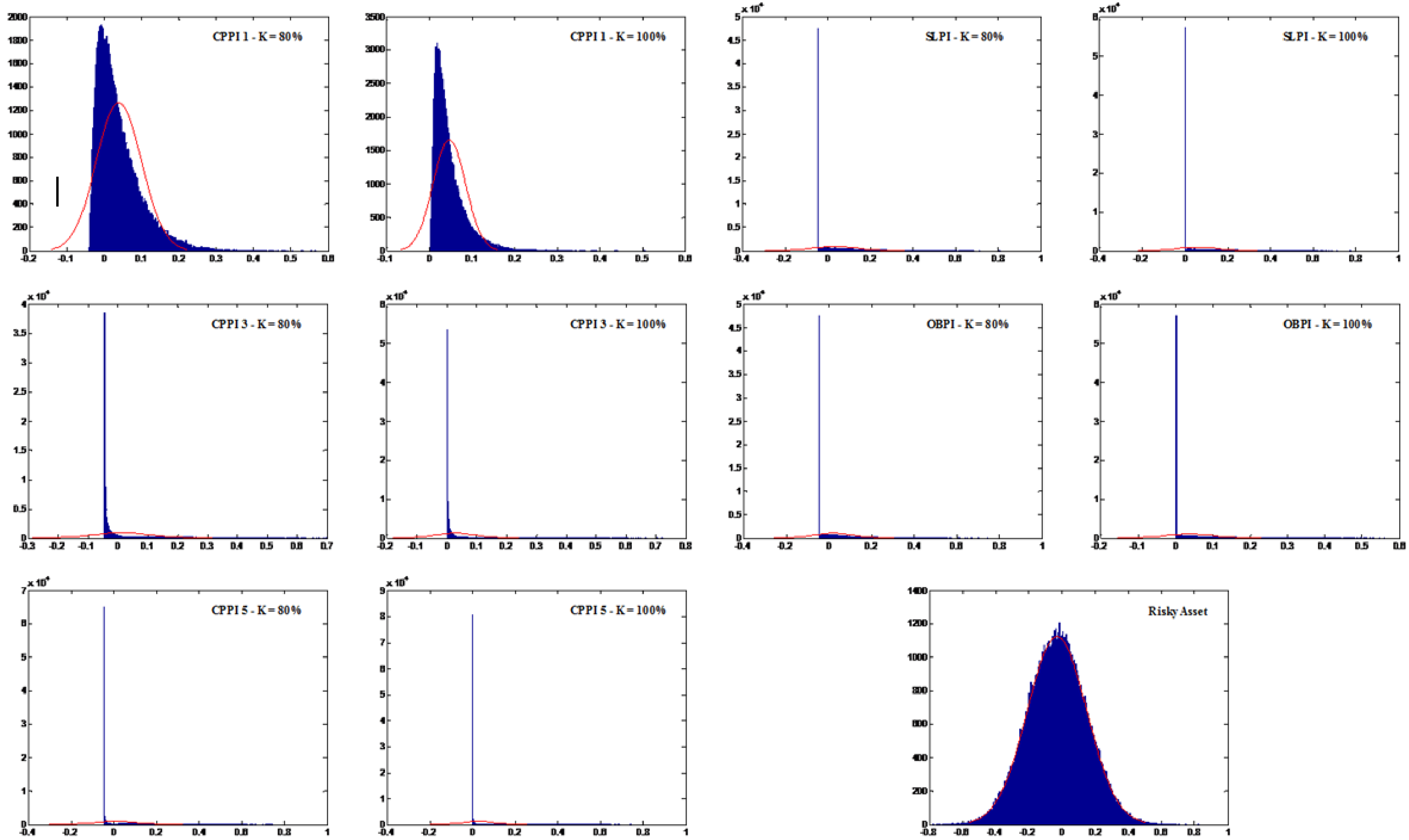
Notes: Probability density function of strategies returns – CPPI 1, CPPI 3, CPPI 5, SLPI, OBPI and Risky Asset. These results were achieved based on the following scenario: μ = -5%; σ = 40 %; r = 5%; T = 5; d = 252; N = 100.000. For each strategy it was assumed a K = 80% and a K = 100%.

Fig. A.4. Return Distributions with scenario: μ = -5% and σ = 40%



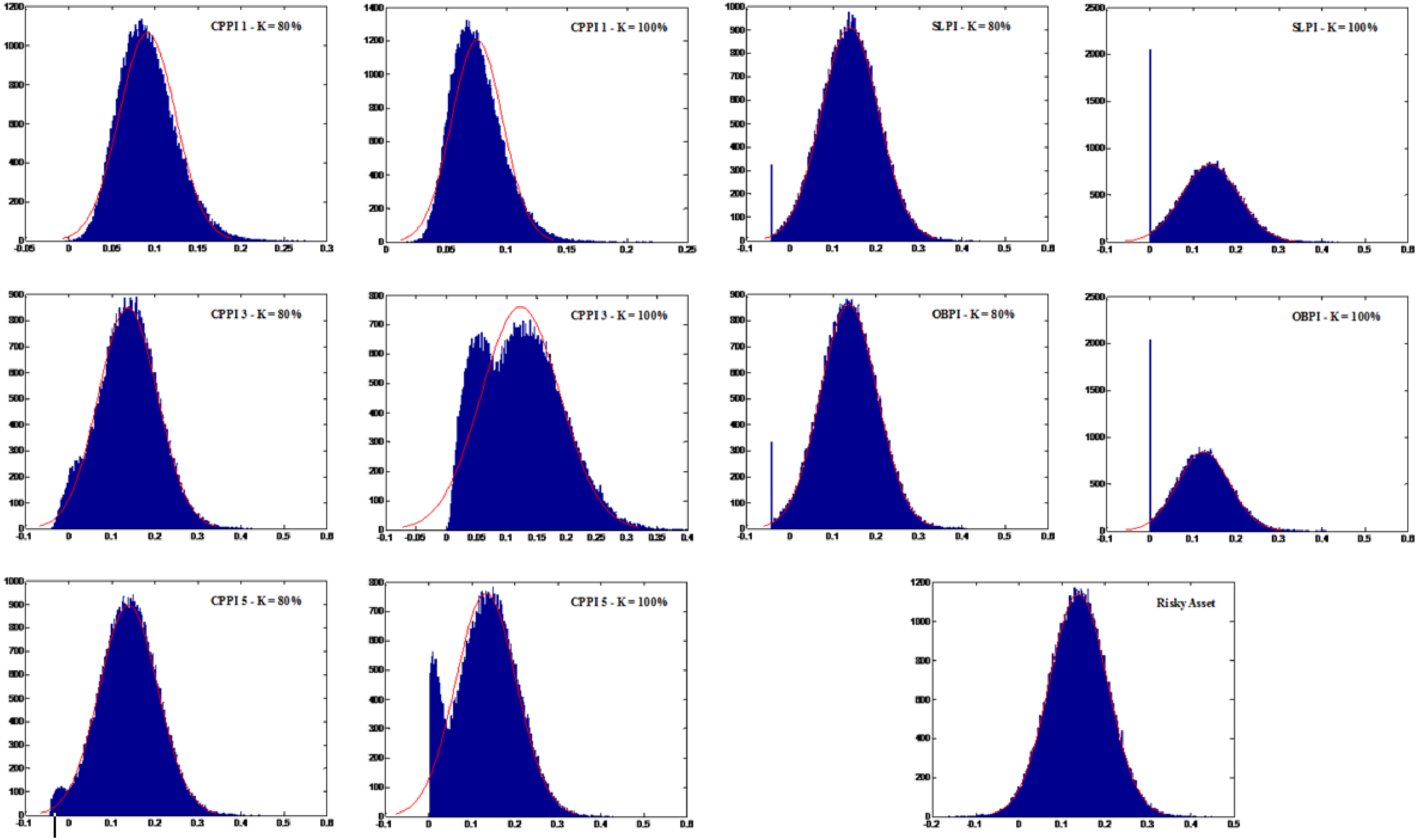
Notes: Probability density function of strategies returns – CPPI 1, CPPI 3, CPPI 5, SLPI, OBPI and Risky Asset. These results were achieved based on the following scenario: $\mu = 5\%$; $\sigma = 15\ \%$; r = 5%; T = 5; d = 252; N = 100.000. For each strategy it was assumed a K = 80% and a K = 100%.

Fig. A.5. Return Distributions with scenario: $\mu = 5\%$ and $\sigma = 15\%$



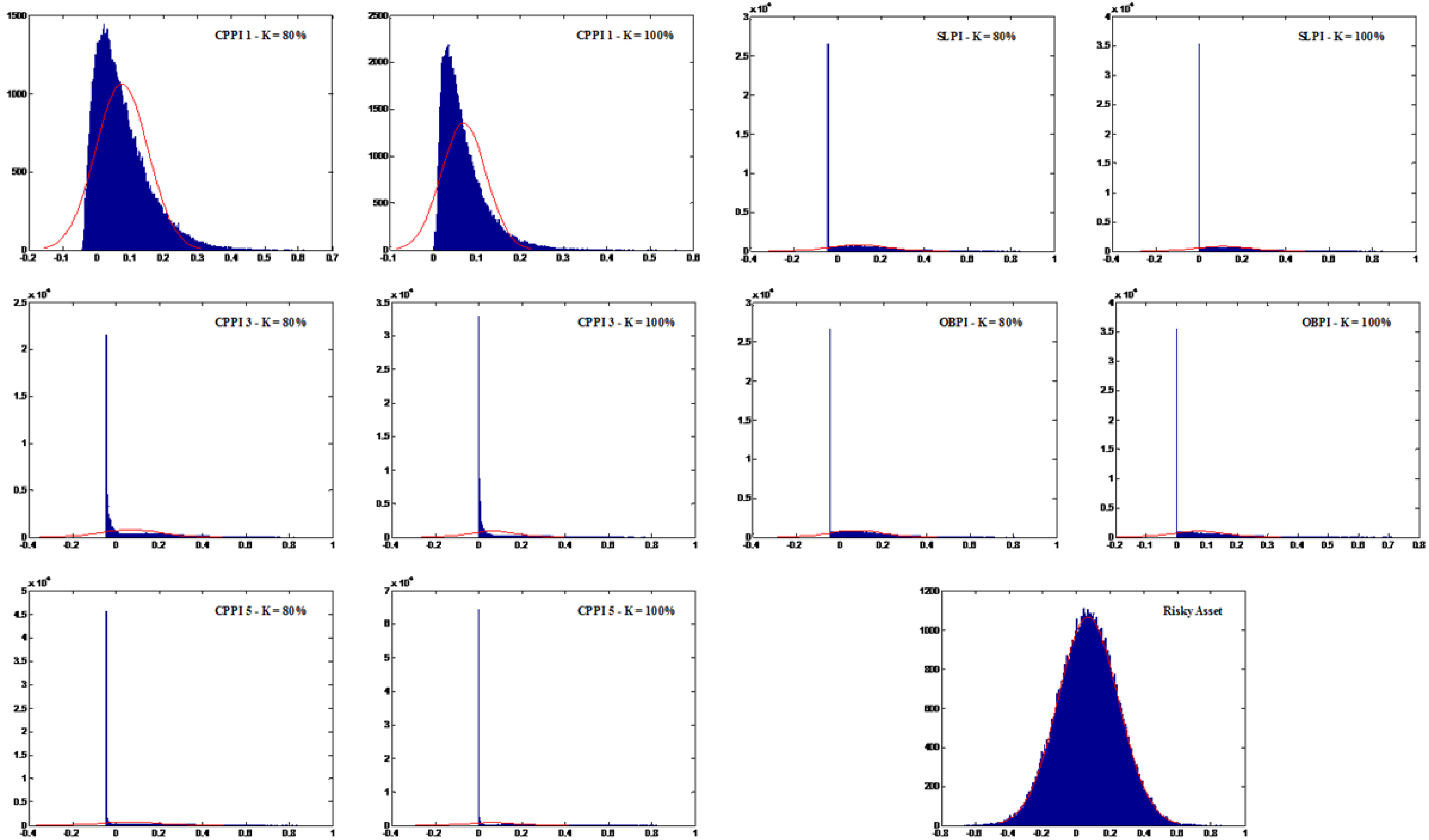
Notes: Probability density function of strategies returns – CPPI 1, CPPI 3, CPPI 5, SLPI, OBPI and Risky Asset. These results were achieved based on the following scenario $\mu = 5\%$; $\sigma = 40\ \%$; $r = 5\%$; $T = 5$; $d = 252$; $N = 100.000$. For each strategy it was assumed a K = 80% and a K = 100%.

Fig. A.6. Return Distributions with scenario: μ = 5% and σ = 40%



Notes: Probability density function of strategies returns – CPPI 1, CPPI 3, CPPI 5, SLPI, OBPI and Risky Asset. These results were achieved based on the following scenario: μ = 15%; σ = 15 %; r = 5%; T = 5; d = 252; N = 100.000. For each strategy it was assumed a K = 80% and a K = 100%.

Fig. A.7. Return Distributions with scenario: μ = 15% and σ = 15%



Notes: Probability density function of strategies returns – CPPI 1, CPPI 3, CPPI 5, SLPI, OBPI and Risky Asset. These results were achieved based on the following scenario: $\mu = 15\%$; $\sigma = 40\ \%$; $r = 5\%$; $T = 5$; $d = 252$; $N = 10.000$. For each strategy it was assumed a K = 80% and a K = 100%.

Fig. A.8. Return Distributions with scenario: $\mu = 15\%$ and $\sigma = 40\%$

Appendix B

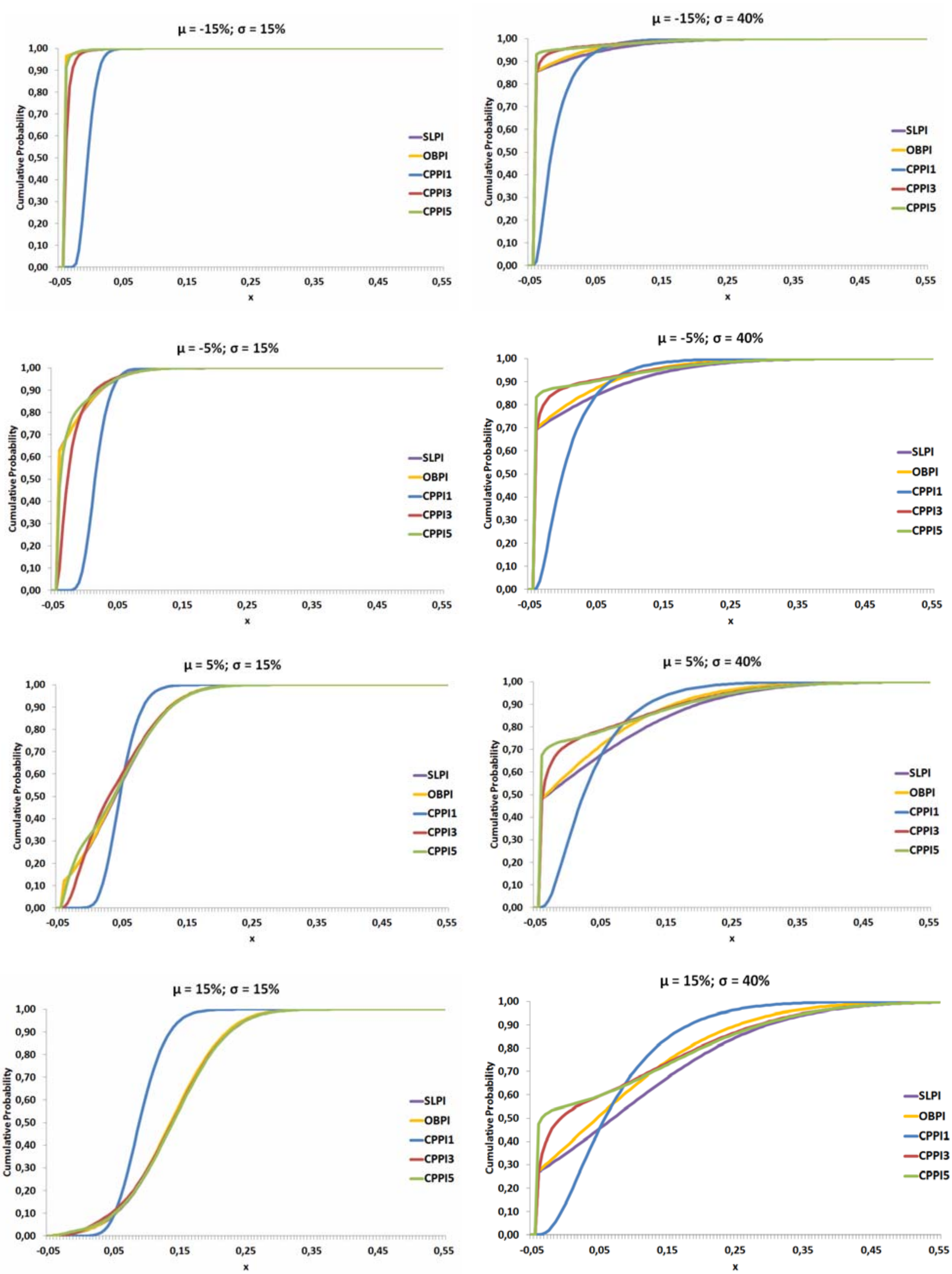


Fig. B.1. Stochastic dominance – first order, with K = 80%

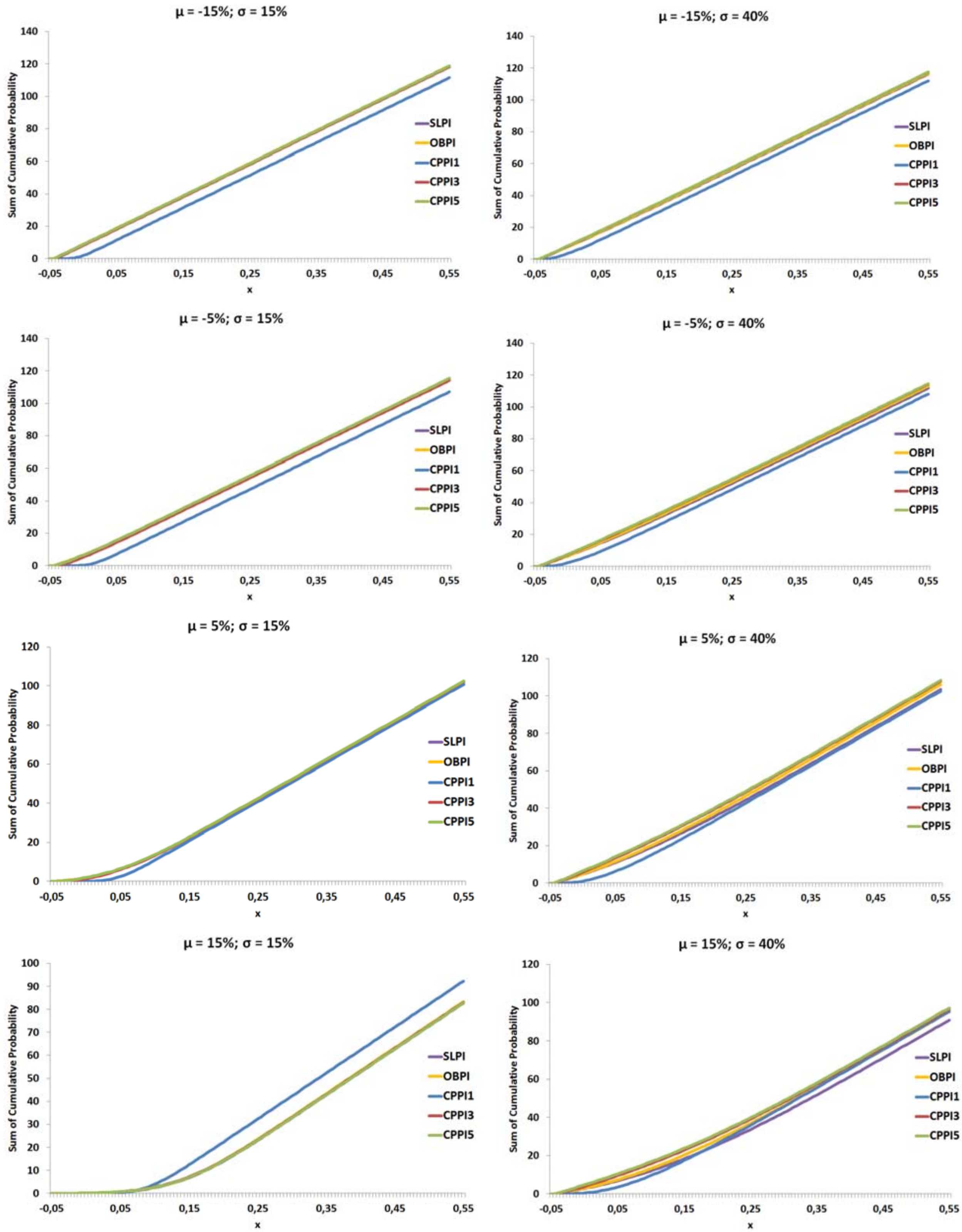


Fig. B.2. Stochastic dominance – second order, with K = 80%

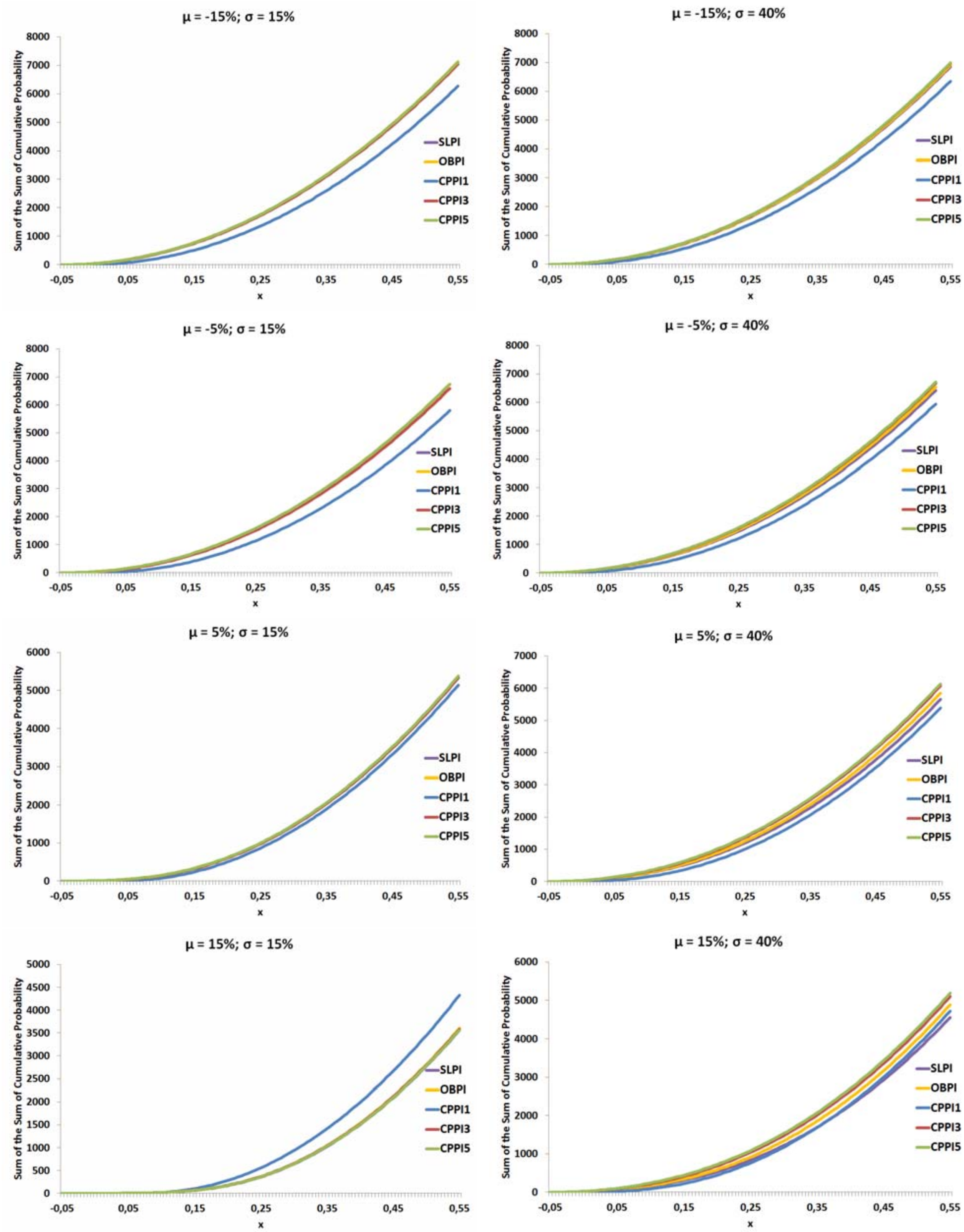


Fig. B.3. Stochastic dominance – third order, with K = 80%

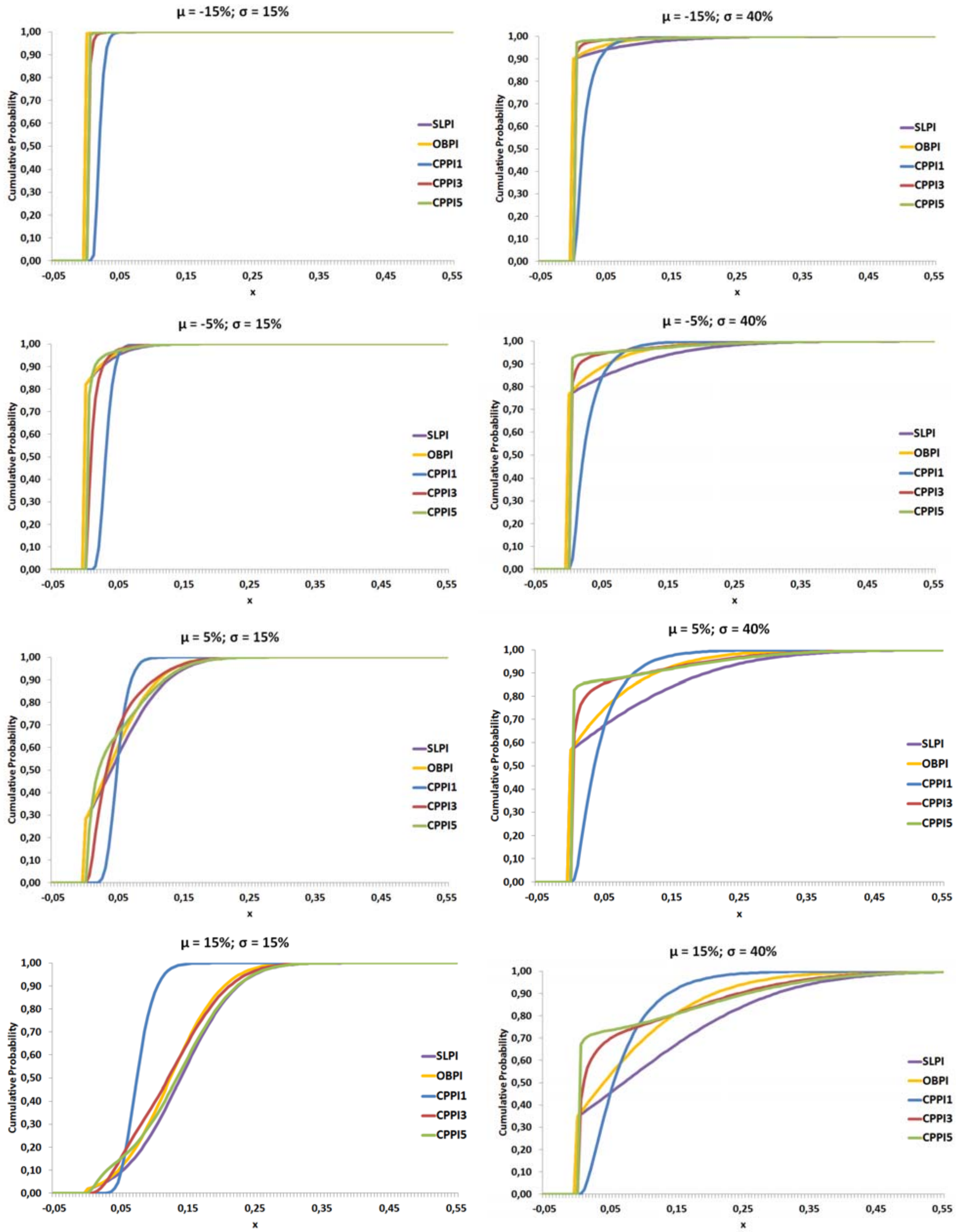


Fig. B.4. Stochastic dominance – first order, with K = 100%

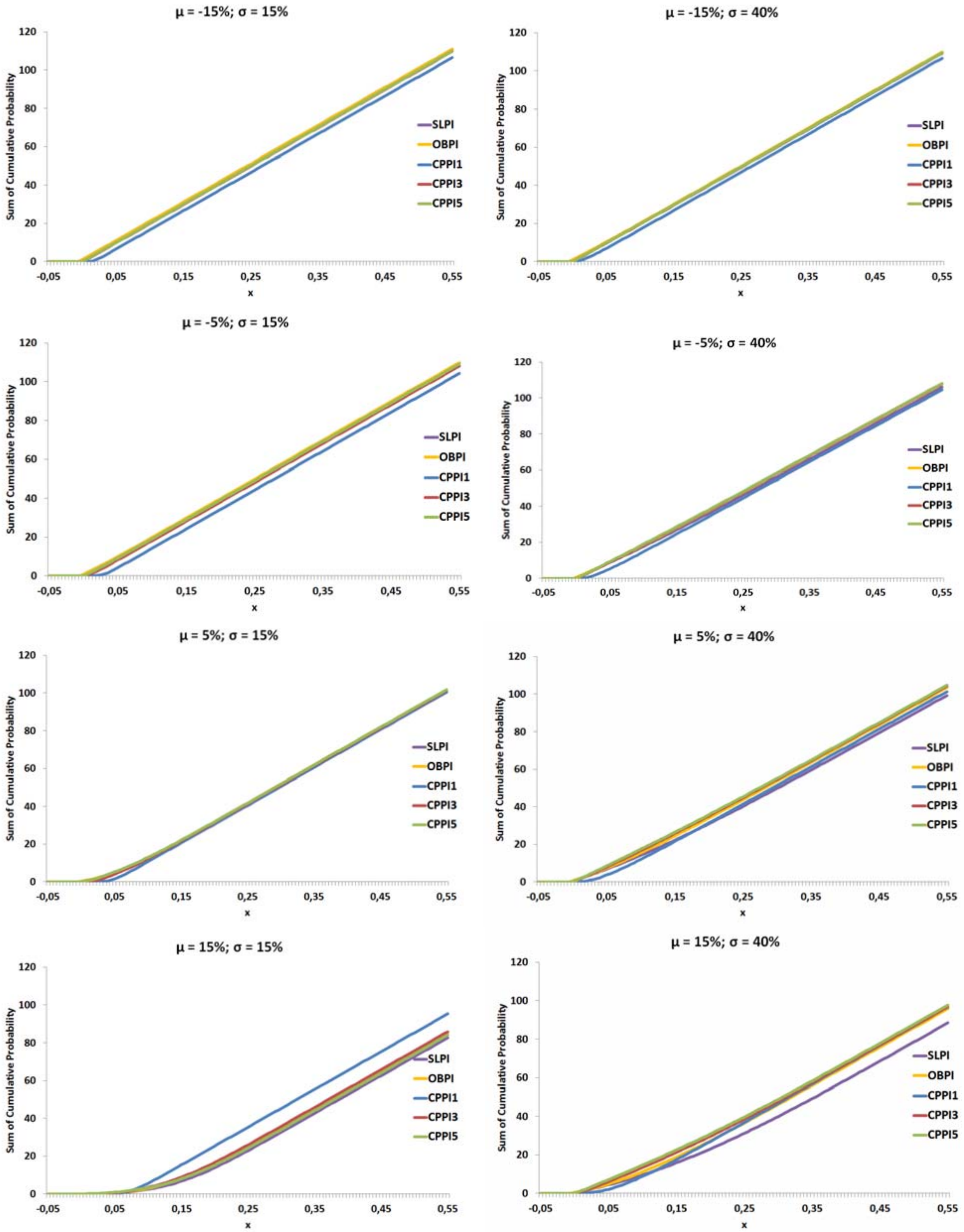


Fig. B.5. Stochastic dominance – second order, with K = 100%

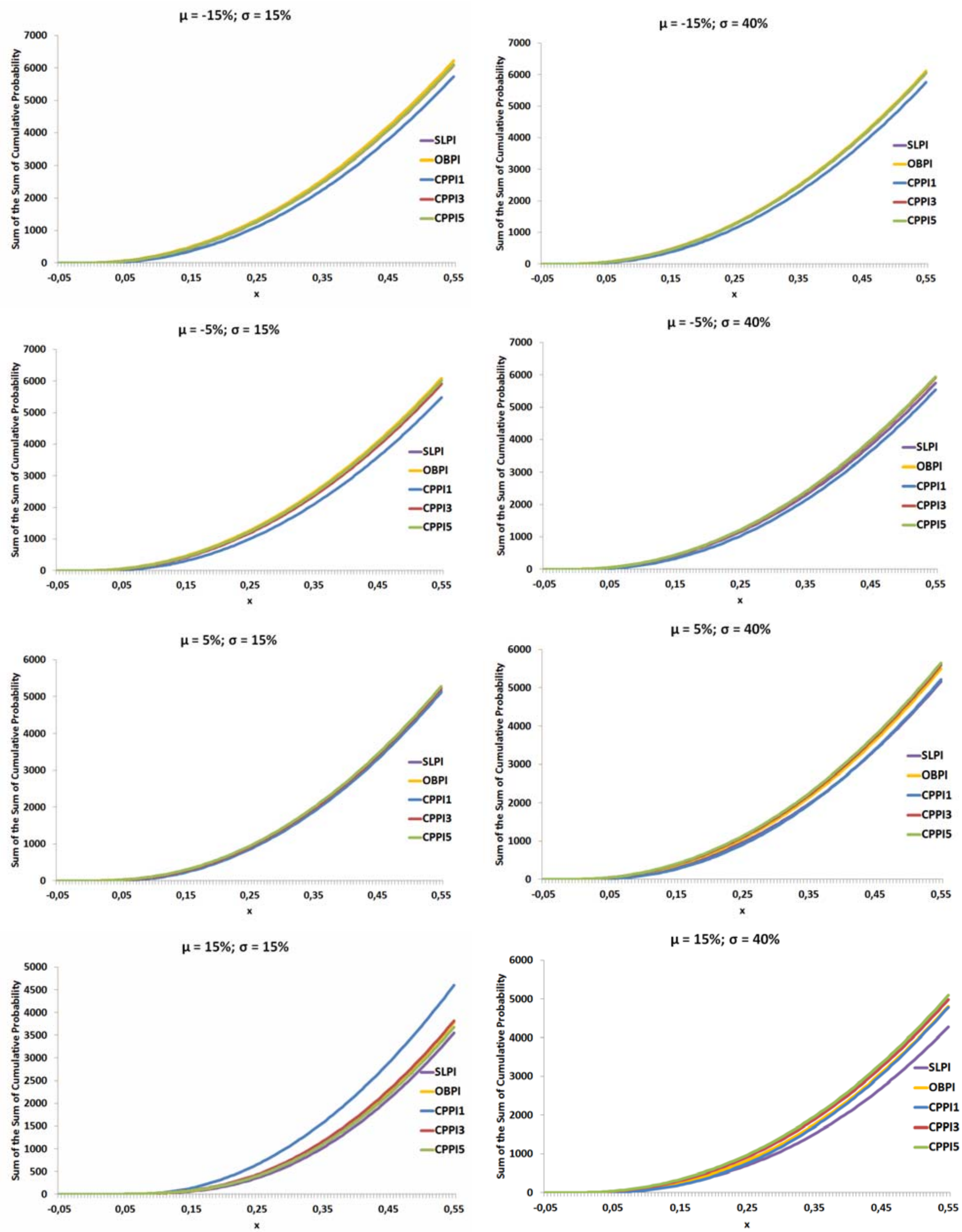


Fig. B.6. Stochastic dominance – third order, with K = 100%