Exercises

1. Plot the density function of the Gamma r.v. with parameters
   a. $\alpha = 1$, $\theta = 1$
   b. $\alpha = 2$, $\theta = 1$
   c. $\alpha = 3$, $\theta = 1$
2. Plot the density function of the Lognormal r.v. with parameters
   a. $\mu = 0$, $\sigma = 1$
   b. $\mu = 0$, $\sigma = 5$
   c. $\mu = 0$, $\sigma = 3/2$
   d. $\mu = 0$, $\sigma = 1$
   e. $\mu = 0$, $\sigma = 1/2$
   f. $\mu = 0$, $\sigma = 1/4$
3. Plot the density function of the Pareto r.v. with parameters
   a. $\alpha = 1$, $\theta = 1$
   b. $\alpha = 2$, $\theta = 1$
   c. $\alpha = 3$, $\theta = 1$

Exercises

4. Is the function

$$F_X(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ 0.01x & \text{for} \quad 0 \leq x < 50 \\ 0.02x - 0.5 & \text{for} \quad 50 \leq x < 75 \\ 1 & \quad\quad x \geq 75 \end{cases}$$

   a distribution function? Justify.

5. Find the constant K so that

$$f_X(x) = Kx^2, \ -K < x < K,$$

   is a probability density function.

6. If $X \sim N(2,1)$ find $\Pr\{|X-2| < 1\}$

Exercises

7. Given the c.d.f.

$$F_X(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ x^2 + 0.2 & \text{for} \quad 0 \le x < 0.5 \\ x & \text{for} \quad 0.5 \le x < 1 \\ 1 & \quad x \ge 1 \end{cases}$$

   express it as a mixture of the distribution functions of a continuous and a discrete r.v.'s.

8. If X is an exponential distribution with mean 2, find $\Pr\{X < 1 | X < 2\}$

9. A die is cast until a 6 appears. What is the probability that it must be cast more than 5 times?

Exercises

10. Determine the distribution, density and hazard rate function for a survival function of the age at death so that

$$\Pr(X > x) = \begin{cases} 1 - 0.01x & \text{for} \quad 0 \le x < 50 \\ 1.5 - 0.02x & \text{for} \quad 50 \le x < 75. \end{cases}$$

Do you think that this model is reasonable described?

11. A random variable X has density function $f_X(x) = 4x(1+x^2)^{-3}$, $x > 0$. Determine the mode of X.
12. A nonnegative random variable has hazard rate function $h(x) = A + e^{2x}$, $x \ge 0$. Assuming known that $S(0.4) = 0.5$, determine A.
13. Consider a r.v. with distribution function $F_X(x) = 1 - x^{-2}$, $x \ge 1$. Determine the mean, median, mode and hazard rate. Determine the probability that X is greater than 20. Comment.

14. Let (x) denote a life aged x, where $x \geq 0$. Let T_x denote the future lifetime of (x), and is supposed to be a continuous random variable This means that $x + T_x$ is the age-at-death for (x). Let F_x denote the distribution function of T_x, so that

$$F_x(t) = \Pr\{T_x \leq t\},$$

and let S_x be the survival function of T_x, so that

$$S_x(t) = \Pr\{T_x > t\}$$

i.e. $S_x(t)$ represents the probability that (x) survives at least t years. Let T_0 be the future lifetime at birth of the same individual. Show that

Exercises

- a.

$$S_x(t) = \frac{S_0(x+t)}{S_0(x)}$$

(or $S_0(x+t) = S_0(x)S_x(t)$, i.e. the probability of survival from birth to age $x+t$ is the product of surviving from birth to age x, and the probability that having survived to age x, of further surviving to age $x+t$).

b. Denote the force of mortality (hazard rate) at age x, by μ_x. In the class μ_x was defined as the ratio $\frac{f_0(x)}{S_0(x)}$, where f_0 is the density function of T_0.

   i. Show that μ_x can be defined as

$$\mu_x = \lim_{dx \to 0^+} \frac{\Pr\{T_0 \leq x + dx | T_0 > x\}}{dx}.$$

   ii. Show that μ_x can be defined as

$$\mu_x = \lim_{dx \to 0^+} \frac{\Pr\{T_x \leq dx\}}{dx}.$$

- c. Let

$$F_0(t) = 1 - (1 - t/120)^{1/6} \text{ for } 0 \leq t \leq 120$$

Calculate the probability that

  i. a newborn life survives beyhond age 40,
  ii. a life aged 40 dies before age 50,
  iii. a life aged 50 survives beyhond age 65,
  iv. calculate μ_x.