Exercise 48
There are 300 consumers and 2 commodities in an economy. Consumer $i$ has a utility function given by $u_i(x^i_1, x^i_2) = \ln(x^i_1) + \ln(x^i_2)$.
Find the equilibrium prices knowing that in a Walras-equilibrium consumer $i$ obtains the bundle $(10,5)$.

Exercise 49
Consider an exchange economy with 2 consumers. Consumer 1’s utility function is given by $u_1(x^1_1, x^1_2) = (1 + x^2_1)e^{x_1}$ and her endowment is $w_1 = (2,1)$. Consumer 2’s utility function is $u_2(x^2_1, x^2_2) = x^1_2x^2_1$ and endowment is $w_2 = (2,3)$. Determine:
1. the individual demand functions;
2. the excess demand function;
3. the Walrasian equilibrium;
4. the contract curve;
5. the core.

Exercise 50
Consider the classic Robinson Crusoe economy, where all production and consumption are carried out by a single agent. Robinson is endowed with $L$ man-hours per week. On his island there is only one productive activity, oyster harvesting from an oyster bed, and only one input to production, Robinson’s labour ($L$). Robinson derives utility from consuming oysters ($c$) and leisure ($R$). Note that we must have $R = L - L$.
1. Let $x = F(L)$ denote the production function for oysters, where $L$ is the amount of labour used. Assume $F’ > 0$, $F'' < 0$, and $F’(0) = -\infty$. Let $u(c, R)$ be Robinson’s utility function, with

$$\frac{\partial u}{\partial c} > 0, \frac{\partial u}{\partial R} > 0, \frac{\partial^2 u}{\partial R^2} < 0, \frac{\partial^2 u}{\partial c^2} < 0, \frac{\partial^2 u}{\partial c \partial R} > 0.$$
a. Determine the efficient allocation in this economy both analytically and graphically;

b. Let market prices for output and labour be \( p > 0 \) and \( w > 0 \), respectively. Find the general equilibrium.

2. Now assume \( F(L) = L^\alpha \), where \( 0 < \alpha < 1 \), and \( u(c, R) = c^\beta R^{1-\beta} \), where \( 0 < \beta < 1 \), and find the general equilibrium.

**Exercise 51**

Consider a one-consumer, one-producer economy. Compute the equilibrium prices, consumptions, and profits when the consumer’s utility function is given by \( u(x, y) = \ln(x) + \ln(y) \) - where \( y \) denotes leisure - the production function is \( f(z) = \sqrt{z} \) - where \( z \) denotes labour - and the total endowment of time is 1.

**Exercise 52**

Consider an economy with 2 commodities and 2 consumers, whose utility functions are given by \( u_i(x^1_i, x^2_i) = \sqrt{x^1_i x^2_i}, i = 1,2 \) and endowments are \( w_1 = (1,3) \) and \( w_2 = (3,1) \). Determine the core of this economy.

**Exercise 53**

Consider a three-agent economy where each agent has a utility function \( u(x, y) = xy \) and endowments are \( w_1 = w_2 = (1,4) \) and \( w_3 = (27,1) \). Show that the following allocation belongs to the core: \( (x_1, y_1) = (6,6) \), \( (x_2, y_2) = (7,7) \), and \( (x_3, y_3) = (16,16) \).

**Exercise 54**

V, Ex. 18.2, p.357.