



School of Economics and Management

TECHNICAL UNIVERSITY OF LISBON

Department of Economics

Tanya Araújo and Francisco Louçã

***Bargaining Clouds, or Mathematics as a Metaphoric
Exploration of the Unexpected***

WP 27/2008/DE/UECE

WORKING PAPERS

ISSN N° 0874-4548



Bargaining Clouds, or Mathematics as a Metaphoric Exploration of the Unexpected

Tanya Araújo * and Francisco Louçã
ISEG (School of Economics and Management)
Technical University of Lisbon (TULisbon)
Rua do Quelhas, 6 1200-781 Lisboa Portugal
Research Unit on Complexity in Economics (UECE) †

April 29, 2008

1 Introduction

In the very first page of his highly regarded novel, *One Hundred Years of Solitude*, Gabriel Garcia Marquez writes that, when arriving at Macondo and discovering so many unknown objects, Aurelio Buendia had to point out these things because no words were defined for them. This metaphor of the process of metaphorisation is an apt description of the scientific process itself, as science points out to what it ignores: denotation generates connotation. Even when science is defined as a self-contained logic, as mathematics once presumed to be, it dares into the territories of the unknown and of the unexpected; the more rigorous, the more daring it ought to be.

Yuri Manin [1], in the paper "Mathematics as Metaphor", commented precisely on this metaphoric quality of mathematics:

"Considering mathematics as a metaphor, I want to stress that the interpretation of the mathematical knowledge is a highly creative act. In a way, mathematics is a novel about Nature and

*corresponding author: tanya@iseg.utl.pt

†Supported by FCT Portugal project PDCT/EGE/60193/2004

Humankind. One cannot tell precisely what mathematics teaches us, in much the same way as one cannot tell what exactly we are taught by "War and Peace".

The epic "War and Peace" tells us much about Humankind, as the rigor of mathematics proposes to do. In each case, an exploration into the nature of evolution and change is at stake. The metaphors consequently produced, either as imaginary descriptions or as precise formal models, suggest new interpretations that therefore produce new meanings. Mathematics is semantics.

In particular, complexity - a metaphor of natural and social relations to be precisely analyzed by mathematical methods - defines an approach which is more insightful to understand dynamics than traditional determinism and positivism. In this paper, we argue that this metaphor is powerful enough to suggest new methods to interpret the emergence of new patterns. In the current example, a stochastic geometry and topological technique are used to describe the structural change in the stock market for the last years.

This new method suggests evidence for a transition of regimes in these markets, measures its dynamics and metaphorically provides a graphic description of the ongoing process.

Mathematics for Complex Systems Science

Complex Systems make use of a plenty of metaphorical developments where the interpretation of the mathematical knowledge (the creative process) gives place to at least two different (and apparently conflicting) perceptions of the system.

The description of Complex Systems generally follows one of two strategies:

1. **Simple System with Complex Behaviour:** They are simple systems because they are characterized by having few degrees of freedom. Nevertheless they display unpredictable behaviour: deterministic and yet apparently random.
 - Mathematical tools: non-linear dynamics
 - Ergodic Invariants
 - measure theory,

- algebra,
- set theory

2. **Complex Systems with Simple collective Dynamics** : They are complex systems because they are characterized by having many degrees of freedom. Nevertheless their collective dynamics display patterns that can be observed at different levels. These patterns usually obey to Scale Laws, giving place to the emergence of simple structures that contrast to the huge amount of complexity that characterizes the individual components of the system.

- Mathematical tools: creation of structures and self-organization
 - Ergodic Invariants
 - measure theory,
 - algebra,
 - set theory,
 - graph theory

An interpretation of the apparently random financial market behaviour would require, as a first step, an incursion in the territory of non-linear dynamics. To this purpose, it is usual to compute some Ergodic Invariants (Lyapunov exponents, entropy measures) as an alternative to traditional modelling of simple stochastic processes.

In the example here considered, we approach the stock market complexity from the Collective Dynamics perspective. To this end, a stochastic geometry technique is used to describe structural change. Topological tools help to complete the picture, and provide a metaphorically approximation through a graphic description of the ongoing market process.

Geometrical and Topological Tools for Complex Markets

Due to their unpredictable behaviour eluding so many established models, stock markets have been widely discussed as an example of complex systems. As a result of such efforts, in recent years new methods were suggested in order to describe the dynamics of changes in the behaviour of complex markets. Because the huge amount of available data, some interesting methods are based on empirically oriented and computationally highly demanding approaches.

This paper develops and applies a stochastic geometry technique designed in order to highlight the definition of a simple object that emerges from the collective behaviour of a complex system. In the current case, the market is described according to the evolution of the 253 stocks consistently measured in the S&P500 index for the last 20 years.

The stochastic geometry proceeds to the metaphorical representation of a stock market as a cloud of points in the space. The use of a properly defined distance (computed from the correlation coefficients between stock returns) gives a meaning to geometric and topological notions in the study of the market.

Given that set of distances between points, our geometrical metaphor discusses three semantic questions:

1. Has the cloud a characteristic dimension?

This is, in other words, the embedding question: what is the smallest manifold that contains the set which has obviously many degrees of freedom? Our strategy is based upon the intuition that, if the proportion of systematic information present in correlations between stocks is small, then the corresponding manifold is a low-dimensional entity, which can be described.

2. Has the cloud any typical shape?

Again, the intuition is that, if the cloud is described as a low dimensional object, its dynamics can be observed as the evolution of its form as it is shaped by the occurrence of bubbles and crises.

In portfolio optimization models, when the systematic and unsystematic contributions to the portfolio risk are distinguished, the former is associated to the correlation between stocks (collective structure) and the later to the individual variances of each stock. Consequently, the leading directions obtained from surrogate data may be taken as reference values that represent the characteristic size with which it contributes to the shape of a market whose components were uncorrelated. They correspond to the characteristic size of the individual (isolated) components of the market. On the other hand, the each leading directions obtained from actual data represent the characteristic size of each structure emerging from the dynamics of the market, that is, associated to each leading direction of the market space.

3. Finally, does the evolution of the cloud follow any specific pattern?

Groups of stocks, having their position on the cloud determined by the distance metric, are observed to evolve in a synchronous fashion. The identification of this behaviour suggests the recourse to the application of topological notions, in addition to the geometrical ones.

In a previous work, we used network coefficients to characterize the existence of topological regimes: the clustering coefficient is proved to contain maximal information on such processes of synchronization .

As in other fields of science [2], synchronization in the market plays an important role in the identification of abnormal periods. When the stock market is investigated, synchronization is at the root of the disproportionate impact of public events relative to their intrinsic information content. Paradoxically, this applies to unanticipated public events but also to pre-scheduled news announcements.

A new method is proposed in order to address these three questions.

2 A stochastic geometry to describe the market

The stochastic geometry strategy is simply stated in the following terms [3]:

1. Pick a representative set of N stocks and their historical data of returns over some time interval and, from the returns data, using an appropriate metric, compute the matrix of distances between the N stocks.

The problem is reduced to an embedding problem in which, given a set of distances between points, one asks what is the smallest manifold that contains the set. Given a graph G and an allowed distortion there are algorithmic techniques to map the graph vertices to a normed space in such a way that distances between the vertices of G match the distances between their geometric images, up to the allowed distortion. However, these techniques are not directly applicable to our problem because in the distances between assets, computed from their return fluctuations, there are systematic and unsystematic contributions. Therefore, to extract relevant information from the market, we have somehow to

separate these two effects. The following stochastic geometry technique is used:

2. From the matrix of distances compute the coordinates for the N stocks in an Euclidean space of dimension smaller than N and then apply the standard analysis of reduction of the coordinates to the center of mass and compute the eigenvectors of the inertial tensor.
3. Apply the same technique to surrogate data, namely to data obtained by independent time permutation for each stock and compare these eigenvalues with those obtained in (2), in order to identify the directions for which the eigenvalues are significantly different as being the market characteristic dimensions.

In so doing, we are attempting to identify the empirically constructed variables that drive the market and the number of surviving eigenvalues is the effective dimension of this economic space.

4. From the eigenvalues of order smaller than the number of characteristic dimensions, compute the difference between eigenvalues in (2) with those in (3). The normalized sum of those differences is the index S , which measures the evolution of the distortion effect in the shape of the market space.

For both surrogate and actual data, the sorted eigenvalues, from large to small, decrease with their order. In the surrogate case, the amount of decrease is linear in the order number, proving that the directions are being extracted from a spherical configuration. The display of a uniform and smooth decrease in the values of the sorted eigenvalues is characteristic of random cases and is also experimentally observed when the market space is built from historical data corresponding to a period of business as usual.

From the returns for each stock

$$r(k) = \log(p_t(k)) - \log(p_{t-1}(k)) \quad (1)$$

a normalized vector

$$\vec{\rho}(k) = \frac{\vec{r}(k) - \langle \vec{r}(k) \rangle}{\sqrt{n(\langle r^2(k) \rangle - \langle r(k) \rangle^2)}} \quad (2)$$

is defined, where n is the number of components (number of time labels) in the vector $\vec{\rho}$. With this vector one defines the distance between the stocks k and l by the Euclidian distance of the normalized vectors.

$$d_{ij} = \sqrt{2(1 - C_{ij})} = \|\vec{\rho}(k) - \vec{\rho}(l)\| \quad (3)$$

as proposed in ([4],[5]) with C_{ij} being the correlation coefficient of the returns $r(i), r(j)$.

The fact that this is a properly defined distance gives a meaning to geometric notions and geometric tools in the study of the market. Given that set of distances between points, the question now is reduced to an embedding problem: one asks what is the smallest manifold that contains the set. If the proportion of systematic information present in correlations between stocks is small, then the corresponding manifold will be a low-dimensional entity. The following stochastic geometry technique was used for this purpose.

After the distances (d_{ij}) are calculated for the set of N stocks, they are embedded in R^D , where $D < n$, with coordinates $\vec{x}(k)$. The center of mass \vec{R} is computed and coordinates reduced to the center of mass.

$$\vec{R} = \frac{\sum_k \vec{x}(k)}{k} \quad (4)$$

$$\vec{y}(k) = \vec{x}(k) - \vec{R} \quad (5)$$

and the inertial tensor

$$T_{ij} = \sum_k \vec{y}_i(k) \vec{y}_j(k) \quad (6)$$

is diagonalized to obtain the set of normalized eigenvectors $\{\lambda_i, \vec{e}_i\}$. The eigenvectors \vec{e}_i define the characteristic directions of the set of stocks. The characteristic directions correspond to the eigenvalues (λ_i) that are clearly different from those obtained from surrogate data. They define a reduced subspace of dimension f , which carries the systematic information related to the market correlation structure. In order to improve the decision criterion on how many eigenvalues are clearly different from those obtained from surrogate data, a normalized difference τ is computed:

$$\tau(i) = \lambda(i) + 1 - \lambda'(i) \quad (7)$$

and the significantly different eigenvalues are those to which $\tau(i) > 1/2$.

2.1 Index of the market structure

As market spaces can be described as low dimension objects, the geometric analysis is able to provide crucial information about their dynamics. In previous papers, we developed different applications of this technique, namely for the identification of periods of stasis and of mutation or crashes. Indeed, market spaces tend to contract during crises along their effective dimensions, but each crisis may act differently on specific dimensions. It is in order to capture that distortion, namely the lack of uniformity along the market effective dimensions, that we defined the market structure index, S [5]. Since the largest f eigenvalues define the effective dimensionality of the economic space, at time t , we compute S as:

$$S_t = \sum_{i=1}^f \frac{\lambda_t(i) - \lambda'_t(i)}{\lambda'_t(i)} = \sum_{i=1}^f \frac{\lambda_t(i)}{\lambda'_t(i)} - 1 \quad (8)$$

where $\lambda_t(1), \lambda_t(2), \dots, \lambda_t(f)$ are the largest f eigenvalues of the market space and $\lambda'_t(1), \lambda'_t(2), \dots, \lambda'_t(f)$ are the largest f eigenvalues obtained from surrogate data, namely from data obtained by independent time permutation of each stock. In computing S , at a given time t , both λ_t and λ'_t are obtained over the same time window and for the same set of stocks.

3 Results and discussion

The application of this measure allows for a description of the evolution of the market for the period under consideration, identifying the shock waves of crashes and measuring their impacts.

Furthermore, considering the lack of uniformity among the market effective dimensions, we are able to characterize the extent to which crashes act differently on specific directions, causing changes in the shape of the market space. The strategy of this measure is thus to identify the distortion of the geometric object describing the market as a basis for the analysis of its dynamics [6].

Looking for relevant distortions in the shape of the S&P500 market space through the last years, we found that amongst the highest values of the index are those computed for some moments of turbulence, such as 19th October 1987, 27th October 1997 and 11th September 2001, as expected (Fig. 1). These crashes can be classified according to a seismography measuring their impact and characteristics [7].

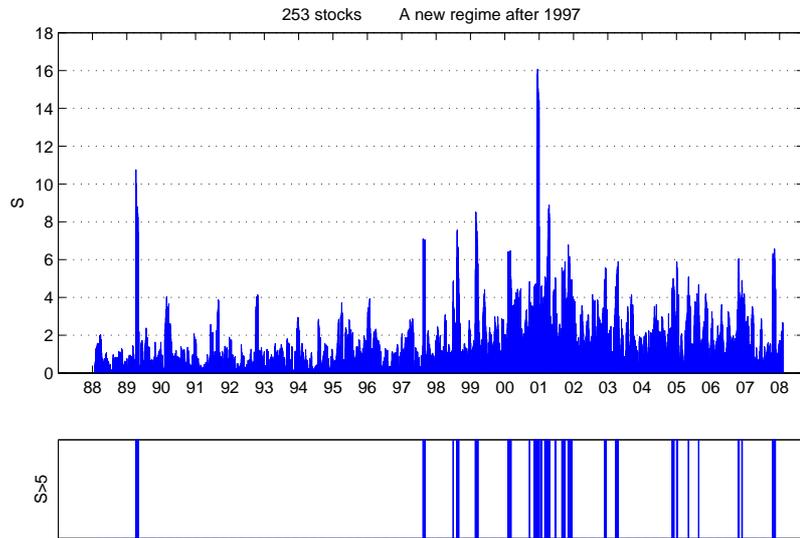


Figure 1: Chronological concentration of crashes in the period under consideration

The index provides information on the evolution of the cloud describing the dynamics of the markets. It indicates the moments of perturbations, proving that the dynamics is driven by shocks and by a structural change. These changes are interpretable as a general dynamics, as suggested by Figures 5 and 6, which demonstrates how crashes cluster in the turn of the century period and since then. But they are also interpretable as a differential sectoral dynamics, since the market is also driven by speculative processes which are fuelled by institutional decisions (such as changes in the reference interest rate, for regulatory reasons) just as it is driven by the spell of differential return and profit rates across sectors.

Indeed, the period after 1995, the Internet Boom, is marked both by the low interest rates as by the high profitability of the ICT stocks and, contrary to the popular description, it is a period of dense transformations and higher risks in the stock market. A new regime is emerging, and it is a regime of frequent storms, to proceed with our metaphor: the cloud is and eventually will frequently distorted. In this sense, this period combines some of the features of the 1920s and some of those characteristics of the manias for canals and railways, which accompanied earlier waves of technical change in

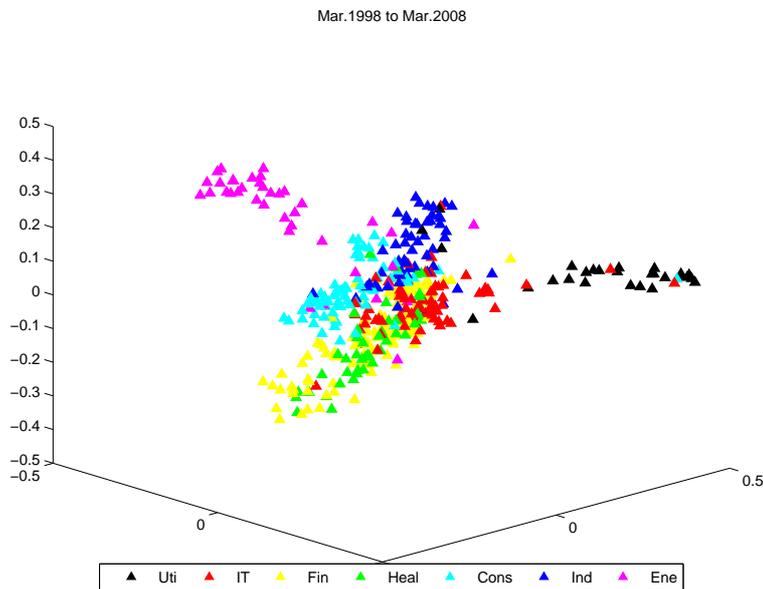


Figure 2: 3-dimensional representation of a turbulent period

the nineteenth-century.

Considering the cumulative history of the index of market structure, it emerges that a major change is occurring since around 1997, imposing a new dynamic structure.

Reconsidering our three questions, an answer is provided by this empirical approach. The cloud has a characteristic dimension, which allows for a description projecting its typical shape and identifying the patterns of its evolution. The index S is useful for this identification of shape and patterns and defines our research, as it is derived from a system of measure and it is part of the logic of a defined mathematics. The conception of the Collective Dynamic of these object, the metaphor implicit in our method, is part of the effort to point out those things we still ignore, as the late Colonel Aurelio Buendia would say.

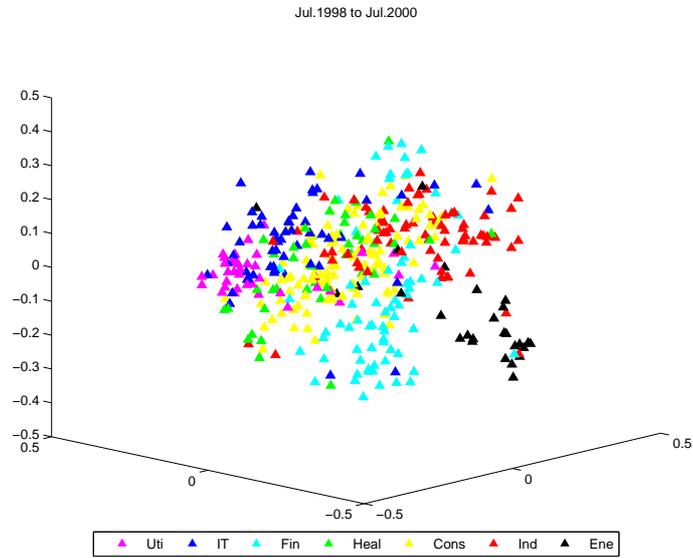


Figure 3: 3-dimensional representation of a business-as-usual period

References

- [1] Manin, Yuri I. 1991, Mathematics as Metaphor, Proceedings of the International Congress of Mathematicians, Kyoto, Japan, 1665-1671
- [2] Stanley, EH, 2000, H. E. Stanley, Exotic Statistical Physics: Applications to Biology, Medicine, and Economics, Physica A 285, 1-17.
- [3] R. Vilela Mendes, T. Araújo and F. Louçã (2003), “Reconstructing an Economic Space from a Market Metric”, Physica A, 323, 635-50.
- [4] Mantegna R.N. (1999), Hierarchical structure in financial markets, European Physics Journal B 11.
- [5] Mantegna, R.N., Stanley, H.E. (2000), An Introduction to Econophysics: Correlations and Complexity in Finance, Cambridge, Cambridge University Press.
- [6] Araujo T, Louçã, F, 2007, The Geometry of Crashes - A Measure of the Dynamics of Stock Market Crises, Quantitative Finance, Vol. 7, No. 1, February 2007, 6374

- [7] Araujo T, Louçã, F, 2008, The Seismography of Crashes in Financial Markets Physics Letters A, Volume/issue: 372/4 pp. 429-434.