4 The new Keynesian model
4.3 Solving the New keynesian model

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Monetary policy in the New keynesian model

Let us start with the three equation model:

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t \]  
\[ \tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - r^n_t) \]  
\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \nu_t \]
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This can be reduced to a two equation first difference model:

\[
\begin{bmatrix}
\tilde{y}_t \\
\pi_t
\end{bmatrix}
= A_T \begin{bmatrix}
E_t\tilde{y}_{t+1} \\
E_t\pi_{t+1}
\end{bmatrix}
+ B_T(\hat{r}_t^n - \nu_t)
\] (4)

where \( \hat{r}_t^n = r_t^n - \rho \) and:

\[
A_T = \Omega \begin{bmatrix}
\sigma & 1 - \beta\phi_\pi \\
\sigma\kappa & \kappa + \beta(\sigma + \phi_y)
\end{bmatrix}
\] (5)

\[
B_T = \Omega \begin{bmatrix}
1 \\
\kappa
\end{bmatrix}
\] (6)

and \( \Omega = \frac{1}{\sigma + \phi_y + \kappa\phi_\pi} \)
The Taylor principle implies that the two eigenvalues of $A_T$ are within the unit circle. Bullard and Mitra (2002) show that:

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$$

(7)

is a necessary and sufficient condition for uniqueness.
We are going to assume that the monetary policy shock follows an autoregressive process given by:

\[ \nu_t = \rho \nu_{t-1} + \epsilon_t \]  

(8)

where \( 0 \leq \rho \nu < 1 \)
Solving the model: the method of undetermined coefficients

This method consists in guessing a functional form for the solution. We know from the simple monetary model that a solution of the model is to express the endogenous variables as a function of the structural shocks. Note that one can solve:

\[ mc = (\sigma + \frac{1 + \varphi}{1 - \alpha})y_t^n - \frac{1 + \varphi}{1 - \alpha}a_t - \log(1 - \alpha) \]  

(9)

For \( y_t^n \). Making use of \( mc = -\mu \), one gets

\[ y_t^n = \psi_{ya}a_t + \vartheta^n \]  

(10)

where \( \vartheta = -\frac{(1-\alpha)(\mu - \log(1-\alpha))}{\sigma(1-\alpha) + \varphi + \alpha} > 0 \) and \( \psi_{ya} = \frac{1 + \varphi}{\sigma(1-\alpha) + \varphi + \alpha} \)
We have seen that the natural rate of interest evolves as:

\[ r^n_t = \rho + \sigma E_t \Delta y^n_{t+1} \]  \hspace{1cm} (11)

using the equation for natural output this can be written as:

\[ r^n_t = \rho + \sigma \psi^n_{ya} E_t \Delta a_{t+1} \]  \hspace{1cm} (12)

Thus the natural rate evolution does not depend on monetary policy shocks.
If one assumes no technology shocks then: \( r^n_t = \rho \)
The solution of the model for the purposes of studying monetary policy shocks implies expressing $\tilde{y}_t$ and $\pi_t$ as a function of monetary policy shocks. We already saw that the natural rate does not depend on monetary policy shocks so $\hat{r}_t^n$ is set to zero. By the method of undetermined coefficients we are going to guess that the solution implies $\tilde{y}_t = \psi_{y\nu} \nu_t$ and $\pi = \psi_{\pi\nu} \nu_t$. 
Solving the model: monetary policy shock

Let us replace the guessed solution in the IS and the New Keynesian Phillips curve:

\[
\psi_{yv}v_t = E_t\psi_{yv}v_{t+1} - \frac{1}{\sigma}(i_t - E_t\psi_{\pi v}v_{t+1} - \rho) \tag{13}
\]

\[
\psi_{\pi v}v_t = \beta E_t\psi_{\pi v}v_{t+1} + \kappa \psi_{yv}v_t \tag{14}
\]

and making use of the monetary policy rule:

\[
i_t = \rho + \phi_{\pi} \psi_{\pi v}v_t + \phi_y \psi_{yv}v_t + v_t \tag{15}\]
Solving the model: monetary policy shock

Solving the system one gets:

\[ \tilde{y}_t = -(1 - \beta \rho_y) \Lambda_y v_t \]  \hspace{1cm} (16)
\[ \pi_t = -\kappa \Lambda_y v_t \]  \hspace{1cm} (17)

where \( \Lambda_y = \frac{1}{(1-\beta \rho_y)[\sigma(1-\rho_y)+\phi_y]+\kappa(\phi_\pi-\rho_y)} \)
Solving the model: technology shock

The real rate in deviations from steady state is given by:

\[ \hat{r}_t = \sigma (1 - \rho_v)(1 - \beta \rho_v) \Lambda_v \nu_t \]  \hspace{1cm} (18)

In turn the nominal rate is given by (in deviations from steady state):

\[ \hat{i}_t = \hat{r}_t + E_t \pi_{t+1} = [\sigma (1 - \rho_v)(1 - \beta \rho_v) - \rho_v \kappa] \Lambda_v \nu_t \]  \hspace{1cm} (19)
Solving the model for a technology shock

Assume a shock process of the form:

$$a_t = \rho a_{t-1} + \epsilon_t$$  \hspace{1cm} (20)

we have already seen that:

$$r^n_t = \rho + \sigma \psi^n_{ya} E_t \Delta a_{t+1}$$  \hspace{1cm} (21)

which implies:

$$\hat{r}^n_t = -\sigma \psi^n_{ya} + (1 - \rho_a) a_t$$  \hspace{1cm} (22)
Solving the model for a technology shock

Now assume \( \upsilon_t = 0 \). It can be shown that:

\[
\tilde{y}_t = (1 - \beta \rho_a) \Lambda_a \hat{r}_t^n
\]  

(23)

\[
= -\sigma \psi^h_{ya}(1 - \rho_a)(1 - \beta \rho_a) \Lambda_a a_t
\]  

(24)

and

\[
\pi_t = \kappa \Lambda_a \hat{r}_t^n
\]  

(25)

\[
= -\sigma \psi^h_{ya}(1 - \rho_a) \kappa \Lambda_a a_t
\]  

(26)

where:

\[
\Lambda_a = \frac{1}{(1 - \beta \rho_a)[\sigma(1 - \rho_a) + \phi_y] + \kappa(\phi_\pi - \rho_a)} > 0
\]
Output and employment will be given by:

\[ y_t = \psi_{ya} n (1 - \sigma (1 - \rho_a) (1 - \beta \rho_a) \Lambda_a) a_t \]  \hspace{1cm} (27)

\[ (1 - \alpha) n_t = y_t - a_t \]  \hspace{1cm} (28)
Model calibration

- \( \sigma = 1 \)
- \( \varphi = 1 \)
- \( \alpha = 1/3 \)
- \( \epsilon = 6 \)
- \( \theta = 2/3 \)
- \( \phi_\pi = 1.5 \)
- \( \phi_y = 0.5/4 \)
- \( \rho_a = 0.9 \)
- \( \rho_\psi = 0.5 \)
Impulse responses - technology shock

- Output gap
- Inflation
- Output
- Employment
- Nominal rate
- Real rate

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Impulse responses- monetary policy shock

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