# The inversion of the spatial lag operator in nonlinear models: fast computation and a closed formula approximation*

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**Abstract**

This paper implements a new method to approximate the inverse of the spatial lag operator matrix, applied to the estimation of nonlinear models with a spatially lagged dependent variable using a Generalized Method of Moments approach. This procedure drastically reduces the computational complexity of the inversion of the operator matrix, proving to be especially valuable for large spatial weights matrices. It is based on a first order truncation and high order approximation of the Taylor series expansion. It explores particular features of spatial weights matrices and, simultaneously, considers the eigensystem structure from higher order powers of normalized matrices. As result, a wide range of matrix operations can be approximated without additional computational burden. A closed formula approximation of the true elements of the inverse of the operator matrix is obtained. The procedure is implemented to the estimation of a nonlinear spatial GMM, preserving the nonlinearity of the model and significantly decreasing the amount of time needed for convergence. A Monte Carlo simulation study shows that the nonlinear spatial GMM with approximated inverse operator matrix performs well in terms of bias and mean square error. In addition, the estimator typically exhibits a minimum trade-off between time and unbiasedness within the class of the nonlinear spatial estimators. Finally, the usefulness of the estimator is illustrated with an empirical application. The authors study the spatial spillovers of competitiveness in the Metropolitan Statistical Areas of the United States of America, to assess the effects over the probability of being a major pole of economic development. These effects are also controlled for environmental quality variables.

**Keywords:** Spatial Econometrics, Nonlinear Models, Inverse Spatial Operator

**JEL Classification:** C18, C31, C35

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