REPÚBLICA PORTUGUESA
EDUCAÇÃO

INSTITUTO DE AVALIAÇÃO EDUCATIVA, I.P.

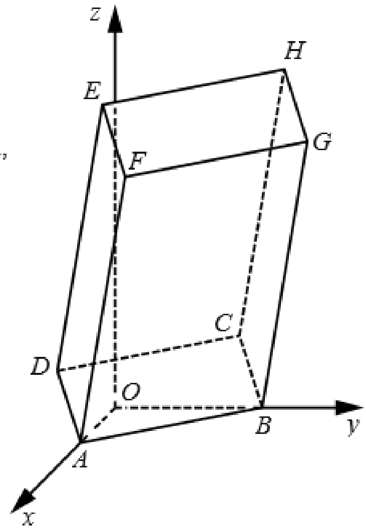
National Final Examination of Mathematics A

Test 635 | 1st Phase | Secondary Schools | 2021

12th Year of Schooling

1. The Figure shows, in an orthonormal frame $Oxyz$ a rectangular parallelepiped $[ABCDEFGH.]$

It is known that:



- Point A belongs to the Ox axis and point B belongs to the Oy axis;
- The coordinates of E and G are $(7,2,15)$ and $(6,10,13)$, respectively.
- The straight line EF is given by $(x,y,z) = (1,-2,19) + k(-3,-2,2),\ k \in \mathbb{R}$.

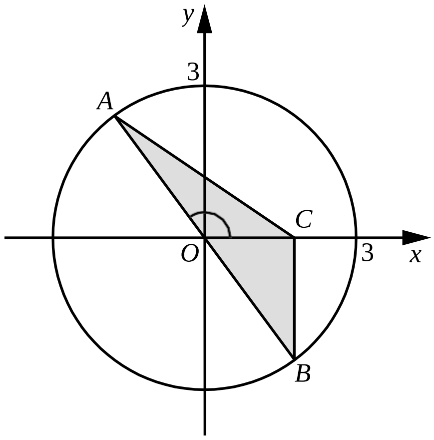
1.1 Which of the following equations defines a line perpendicular to the line $EF$and passing through point E ?

(A) $(x,y,z) = (7,-3,3) + k(2,-3,0),\ k \in \mathbb{R}$.
(B) $(x,y,z) = (7,2,15) + k(0,3,-3),\ k \in \mathbb{R}$.
(C) $(x,y,z) = (7,-10,3) + k(0,3,3),\ k \in \mathbb{R}$.
(D) $(x,y,z) = (7,2,15) + k(2,0,-3),\ k \in \mathbb{R}$.

1.2 Determine, without using a calculator, the reduced equation of the spherical surface with centre at point B and passing through point D.

2. The figure shows, in a orthonormal frame Oxy, the circle with center at O and radius 3. and the triangle ABC.

It is known that:



- The line segment $[AB]$ is a diameter of the circle.
- α is the inclination of AB, $\alpha \in \left]\frac{\pi}{2}, \pi\right[$;
- Point C belongs to the positive semiaxis Ox;
- The line BC is parallel to the Oy axis.

Show that the area A of the triangle $[ABC]$ is given by the expression $A = -9\cos\alpha\sin\alpha$

3. In a school attended by Portuguese and foreign students, 60% of students are girls and 15% are foreign boys.

A pupil from that school was chosen at random and it turned out to be a boy.

What is the probability that he is Portuguese?

(A) 45% **(B)** 50% **(C)** 57,5% **(D)** 62,5%

4. Corfball is a mixed team sport, originating in the Netherlands.

A corfball club from a certain country is going to take part in an international tournament.

The delegation will travel overland, using a five-seater car and a nine-seater van. The delegation is made up of three officials, one coach, five male and five female players.

Write an expression giving the number of different ways of distributing the fourteen members of the delegation among the fourteen available places, knowing that the drivers are two of the officials and that there are twoplayers of each sex in the car.

5. A 11th year class consists of 30 pupils aged 15, 16 and 17, of whom 60% are girls. It is known that one third of the boys are 17 years old and one third of the girls are 15 or 16 years old. André and Beatriz, students in the class, are twins and are 16 years old.

Five students are chosen at random from the class.

Determine the probability that this group consists of André, Beatriz, two17-year-olds and another 15 or 16-year-old.

Present the result as a decimal, rounded to two decimal places.

6. Let (v_n) be a geometric progression. It is known that $v_5 = 4$ and $v_8 = 108$.

What is the value of that v_6 ?

(A) 12 **(B)** 24 **(C)** 48 **(D)** 60

7. Let (u_n) the sequence defined by $u_n = 2 + \frac{(-1)^{n+1}}{n}$.

Determine, without using a calculator, how many odd terms of the sequence belong to the interval $\left[\frac{83}{41}, \frac{67}{33}\right]$

8. In $\mathbb{C}$, the set of complex numbers, we consider $z_1 = 2e^{i\frac{\pi}{4}}$ and $z_2 = 2e^{i\frac{3\pi}{28}}$.

It is known that in the complex plane, the affix of the complex number w is one of the vertices of a regular polygon with center at the origin of the frame and with another vertex on the real positive half-axis.

What is the minimum number of vertices of this polygon?

(A) 7 **(B)** 14 **(C)** 21 **(D)** 28

9. In the set of the complex numbers, consider $z_1 = -3 + 2i$, $z_2 = 1 + 2i$ and $z_3 = 2 - i$.
Let $w = \frac{z_1 z_2}{z_3}$.
Show, without using a calculator, that $|w| = \sqrt{13}$ and $Arg(z) \epsilon \left]-\frac{3\pi}{4}, -\frac{\pi}{2}\right[$.

10. Let f be the function of domain $]0, +\infty[$ defined by

$$f(x) = \begin{cases} x^2\,(1 + 2\ln x) & if\ 0 < x \leq 1 \\ \dfrac{5 - 5e^{x-1}}{x^2 + 3x - 4} & if\ x > 1 \end{cases}$$

Solve 10.1 and 10.2 without using a calculator:

10.1 Check whether f is continuous at $x = 1$.

10.2 Study, in the interval $]0,1[$, the function f regarding the monotony and the existence of local maximums and minimums.

11. Solve this item without using a calculator.

Let g be the function of domain $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ given by $g(x) = x\cos(x) + \sin(x)$. Show, using the Intermediate Value Theorem, that there exists at least one point of the graphic of g such that the tangent straight line to the graphic of g at that point has slope $m = -\frac{1}{2}$.

12. Solve this item without using a calculator.

Let h be the function of domain $\mathbb{R}^+$given by $h(x) = \frac{x^3}{2x^2 - \ln x}$.

Study the existence of a oblique asymptote to the graphic of h, and, if such an asymptote exists, write its reduced equation.

13. Determine, without using a calculator, the real numbers that are solutions of the equation

$$\ln((1-x)e^{x-1}) = x.$$

14. Consider, for a positive real number k, the functions f and g, of domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ given by

$$f(x) = k \sin 2x \text{ and } g(x) = k \cos x.$$

In an orthonormal frame, consider the points A, B and C, the intersections of the graphics of f and g, being A the point of smaller abscissa and B the point of greater abscissa. It is known that the triangle $[ABC]$ is rectangle in B.
Compute the value of k without using a calculator.